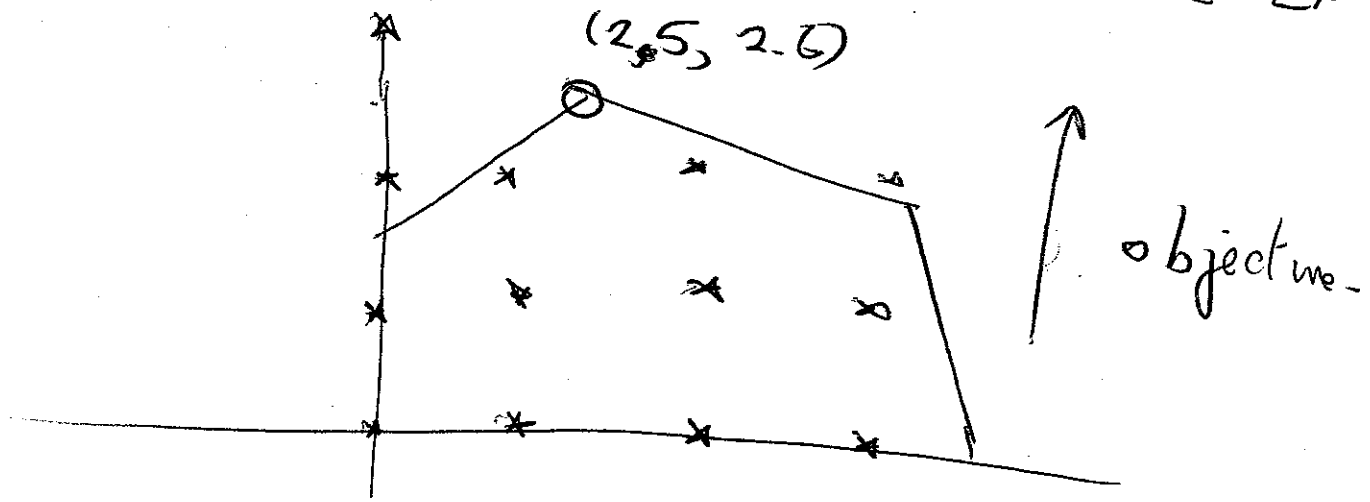
10/9/15

## Branch & Bound

Begins the same way as ~~for~~ cutting algorithm: ie we solve ~~the~~ the LP rela




LP Optimum $x_1 = 2.5$

We split problem into 2 sub-problems

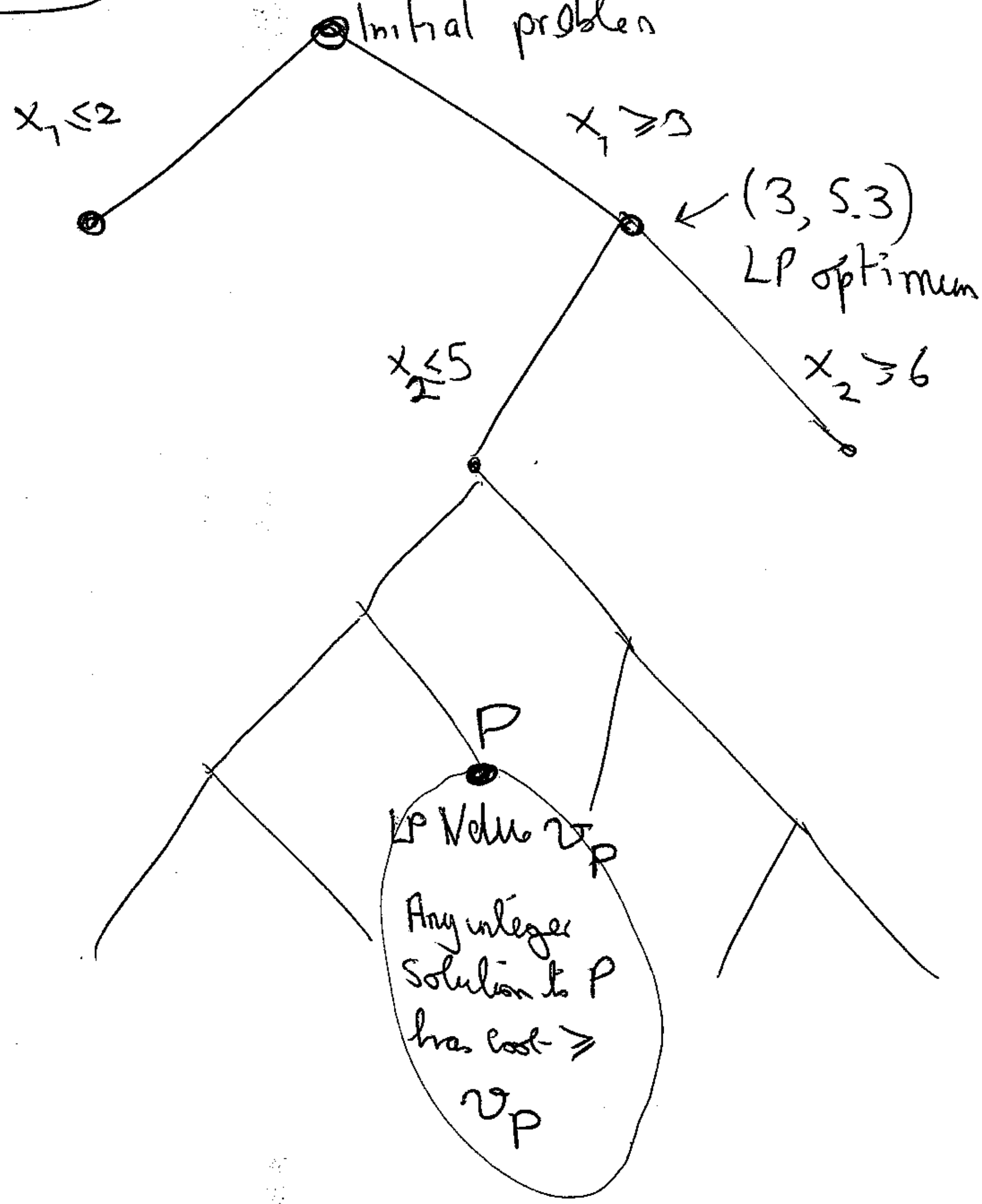
Add $x_1 \leq 2$ in one problem

$x_1 \geq 3$ in other problem

and solve the two subproblems

# Minimization

Problem P




Suppose you already a solution of value $\sigma$.

If $\sigma \leq v_P$, no need to solve P.

Bound part

Similarly if LP is infeasible, no need to explore P.

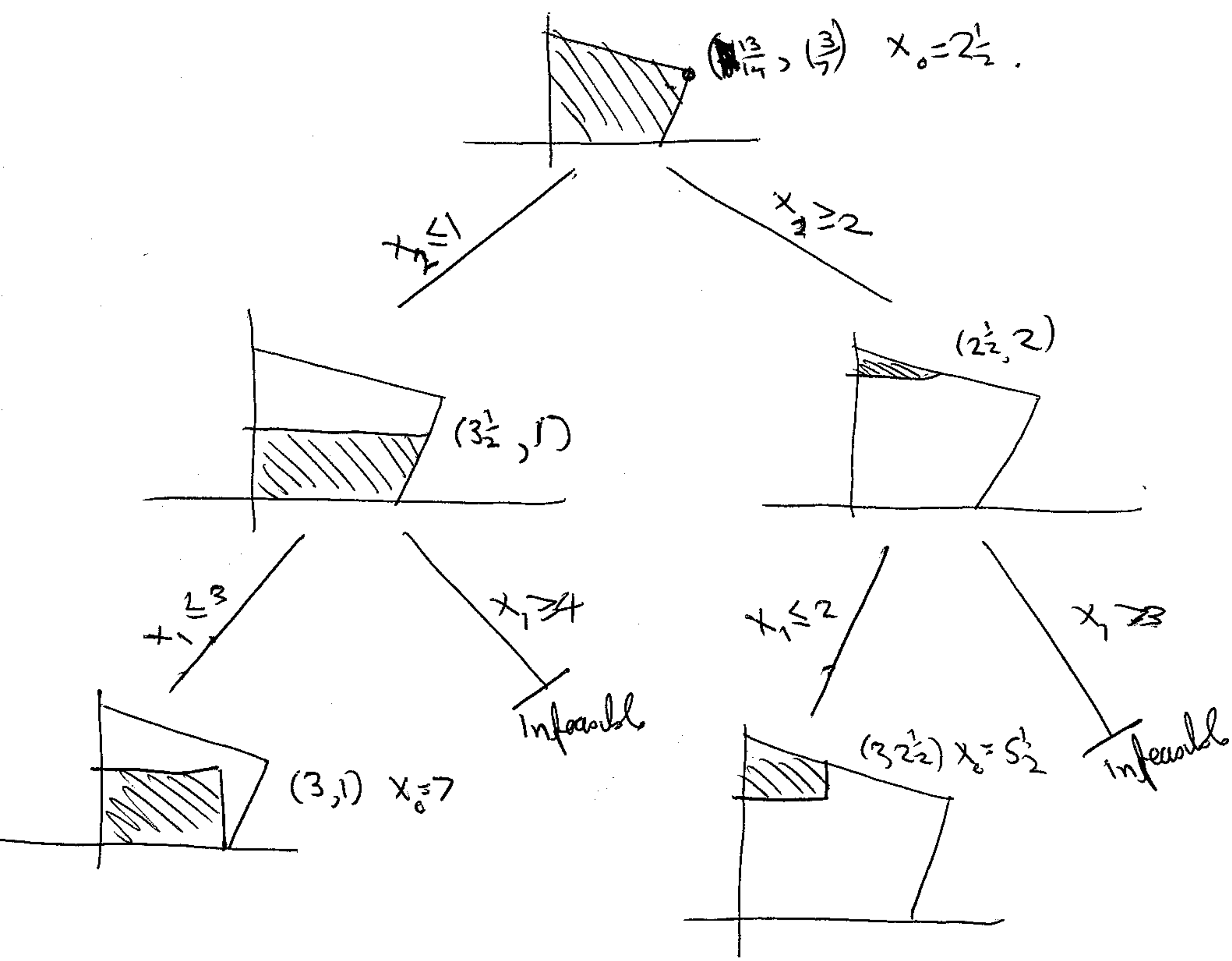
# Example

Minimize $20 - 3x_1 - 4x_2$

s.t. $\frac{2}{5}x_1 + x_2 \le 3$

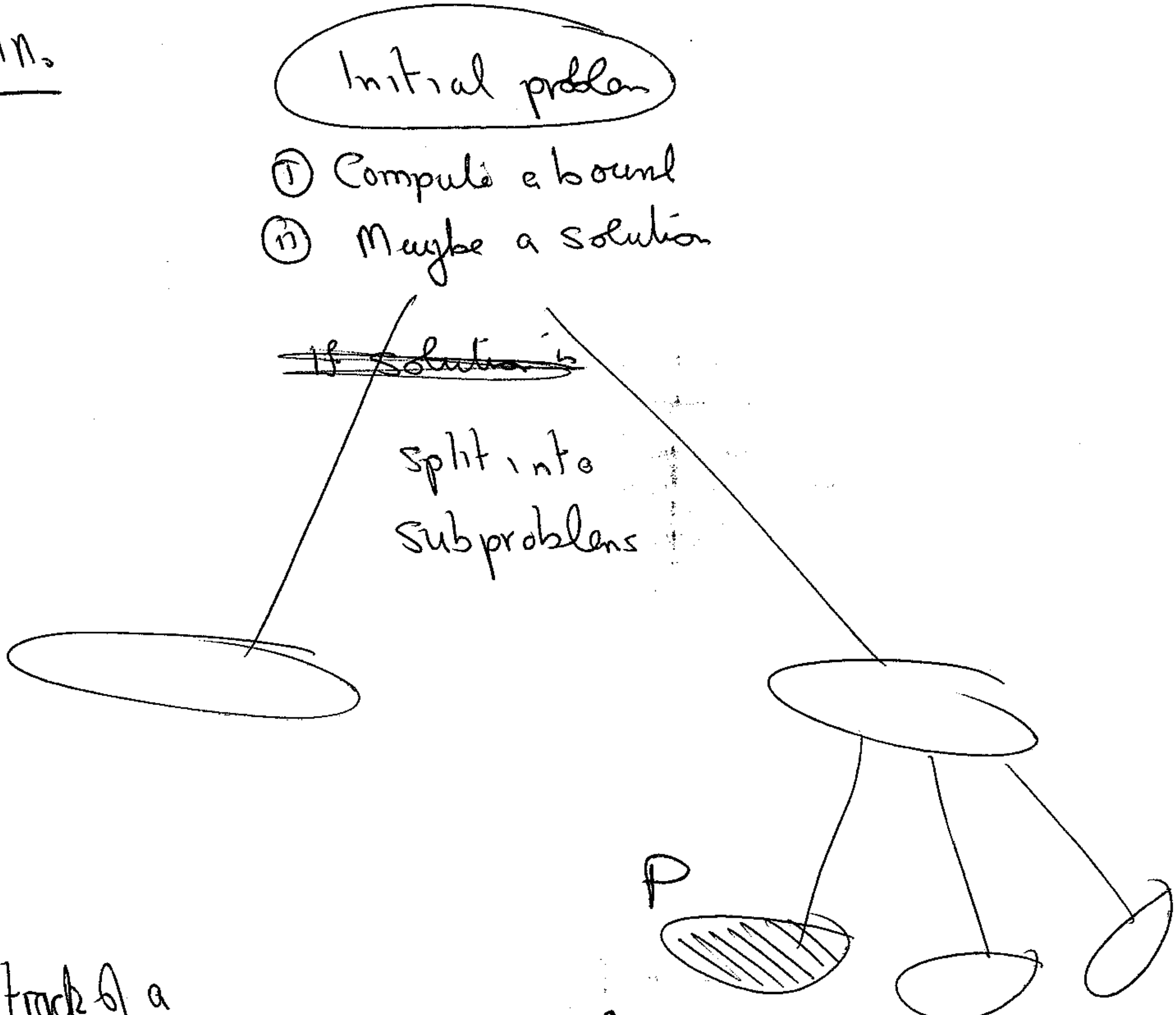
$\frac{2}{5}x_1 - \frac{2}{5}x_2 \le 1$

$x_1, x_2 \ge 0$ & integer



Branch & Bound can be applied to any discrete problem.

<u>Min.</u>




Keep track of a solution with value $\sigma$

Bound bp

if $bp \geq \sigma$ - P is "solved".

# Implicit Enumeration

$$\text{minimize } Z = 7x_1 + 3x_2 + 2x_3 - x_4 - 2x_5$$

$$4x_1 + 2x_2 - x_3 + 2x_4 + x_5 \geq 3$$

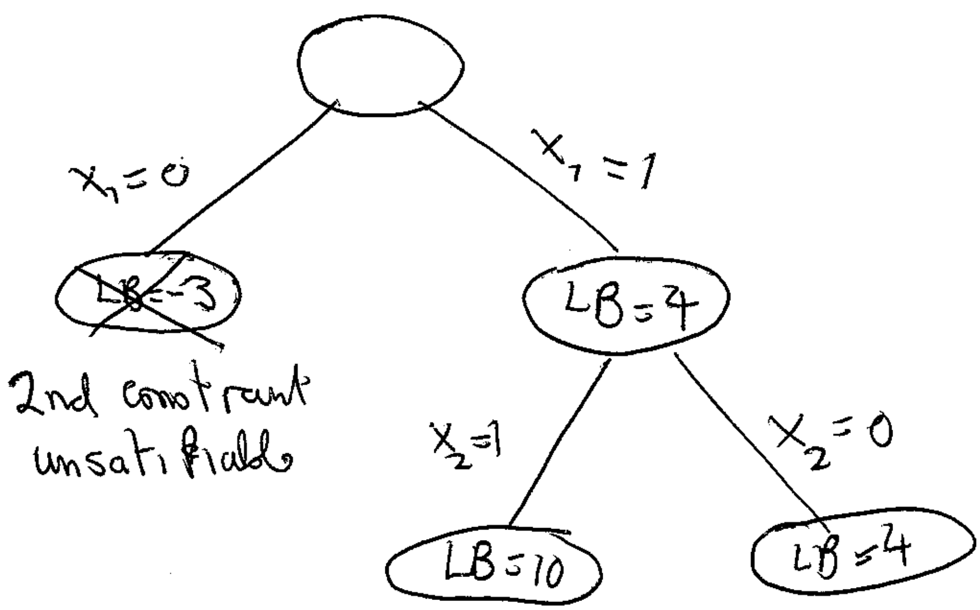
$$4x_1 + 2x_2 + 4x_3 - x_4 - 2x_5 \geq 7$$

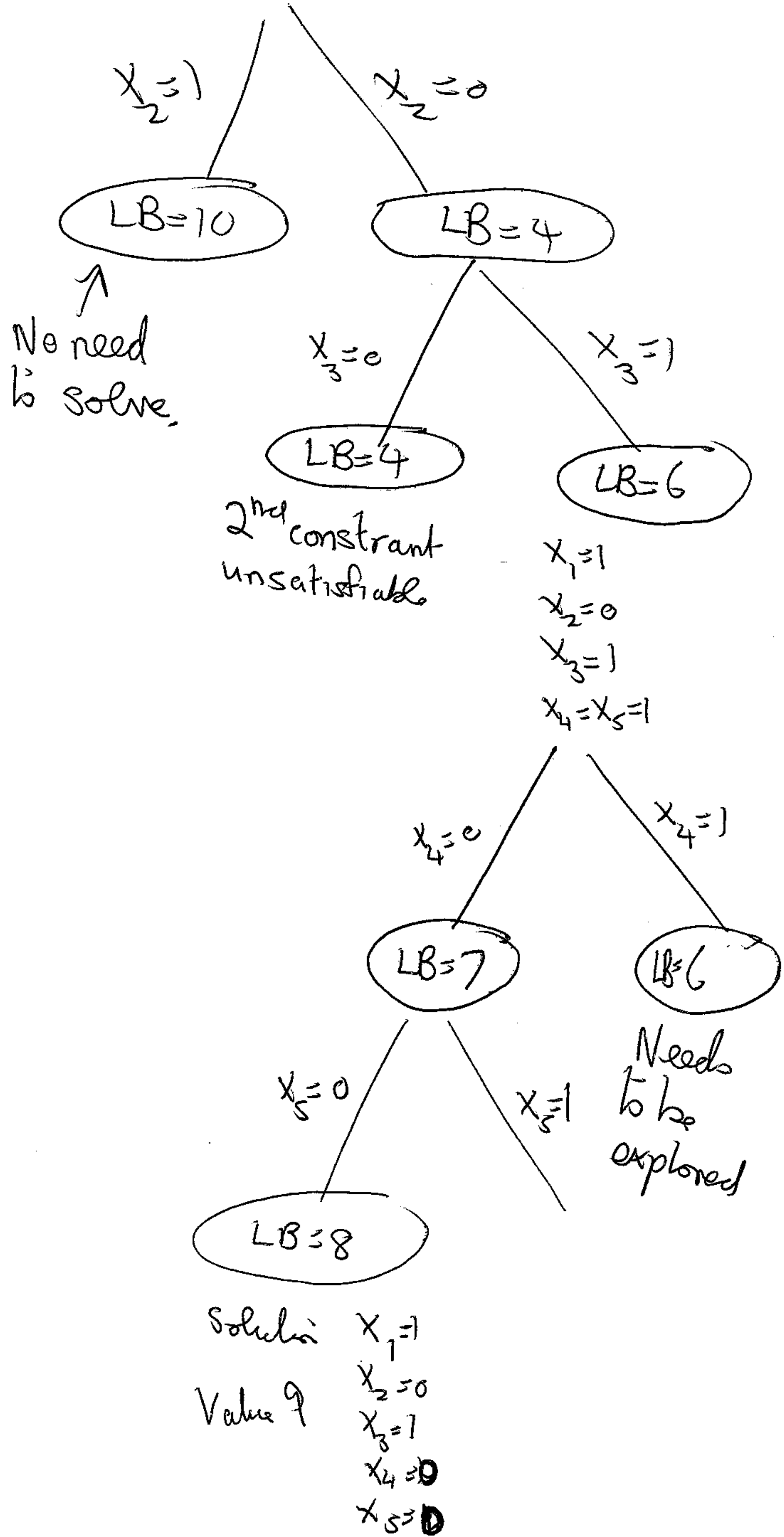
$$x_1, x_2, \ldots, x_5 = 0/1$$

Compute lower bound: $-3$

$x_1 = x_2 = x_3 = 0$
$x_4 = x_5 = 1$
Not feasible.

$x_1 = 0$ or $x_1 = 1$





$x_2=1$
$x_2=0$
LB=10
LB=4
No need to solve.
$x_3=0$
$x_3=1$
LB=4
LB=6
2nd constraint unsatisfiable
$x_1=1$
$x_2=0$
$x_3=1$
$x_4=x_5=1$
$x_4=0$
$x_4=1$
LB=7
LB=6
Needs to be explored
$x_5=0$
$x_5=1$
LB=8
Solution $x_1=1$
Value 9 $x_2=0$
$x_3=1$
$x_4=0$
$x_5=0$