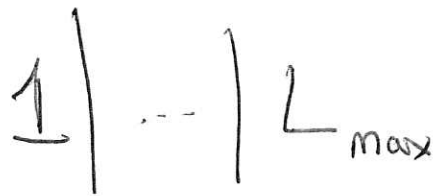


10/26/15

Ex 2



Job  $j$  has a due date  $d_j$

$$L_j = (C_j - d_j)^+$$

$$L_{\max} = \max_j L_j$$

Sort so that  $d_1 \leq d_2 \leq \dots \leq d_n$

Suppose we do not use this order

Old



$$d_k < d_j$$

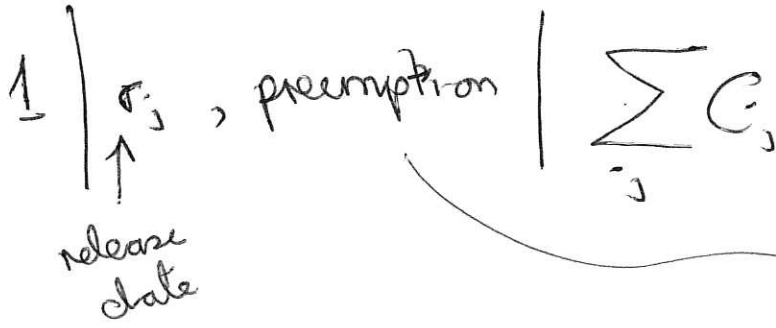
Compare:  $\max((C_j^{\text{old}} - d_j)^+, (C_k^{\text{old}} - d_k)^+) = (C_k^{\text{old}} - d_k)^+ \geq$   
 $\neq \max((C_j^{\text{new}} - d_j)^+, (C_k^{\text{new}} - d_k)^+)$

$$C_k^{\text{old}} = C_j^{\text{new}}$$

New

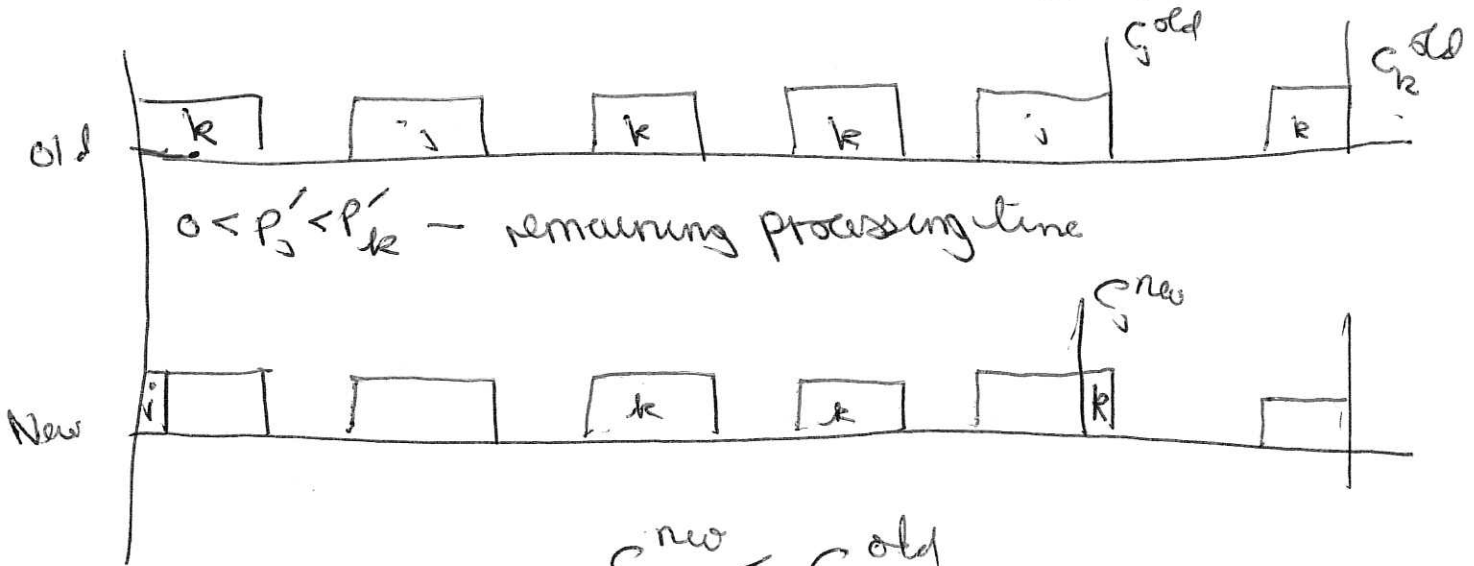


# Ex 3

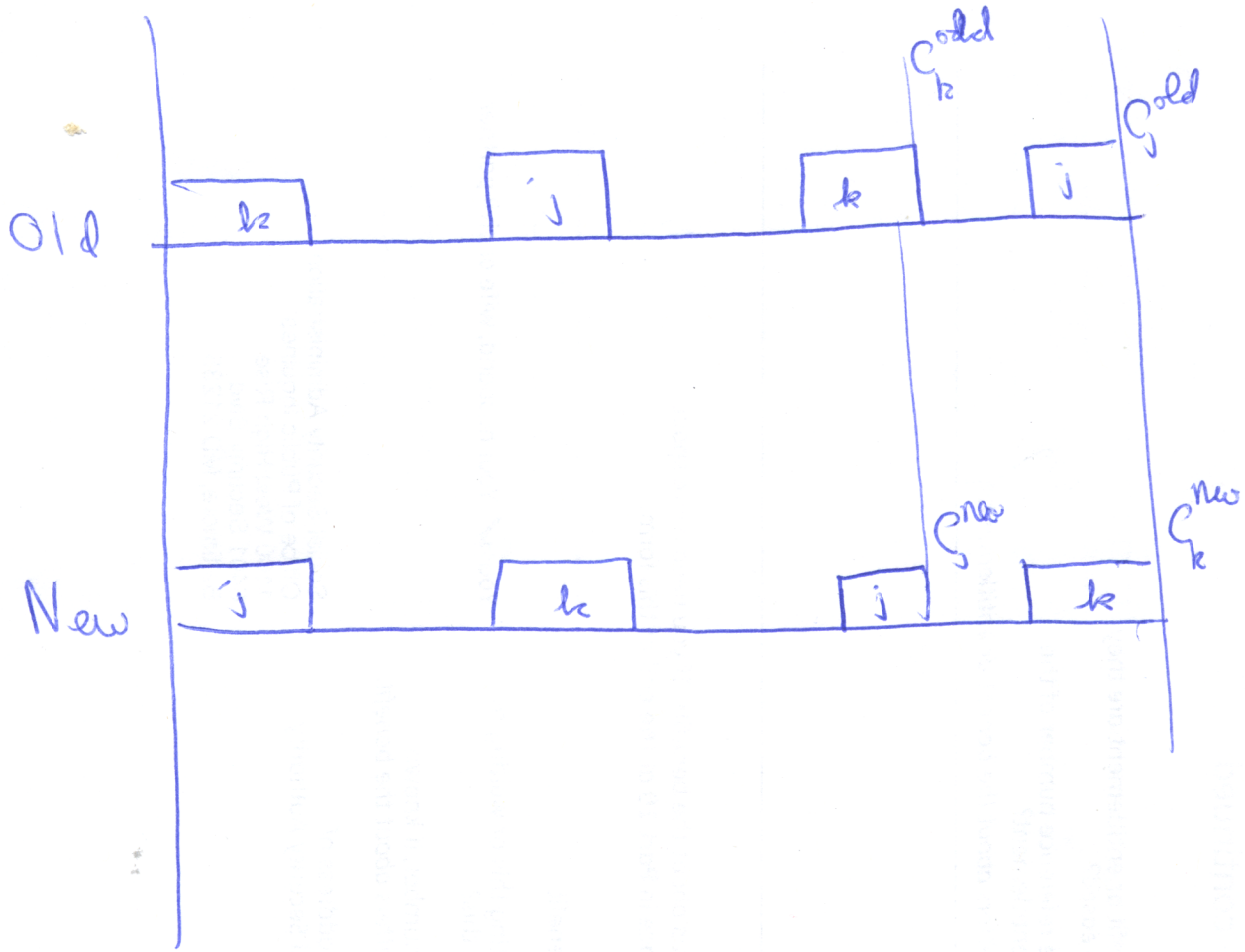


A new job ~~of higher~~  
can eject an old  
job from the  
machine

Algorithm SRPT rule - shortest  
remaining  
processing  
time

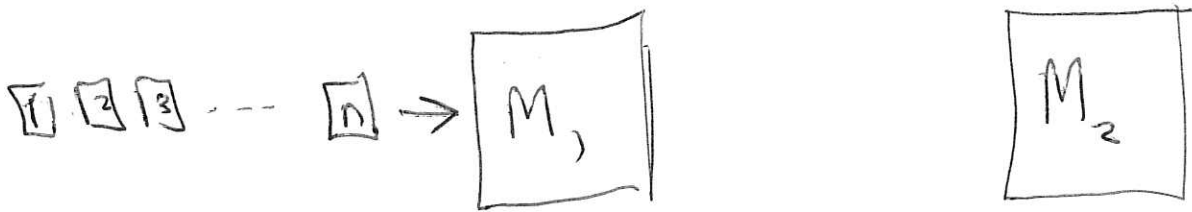
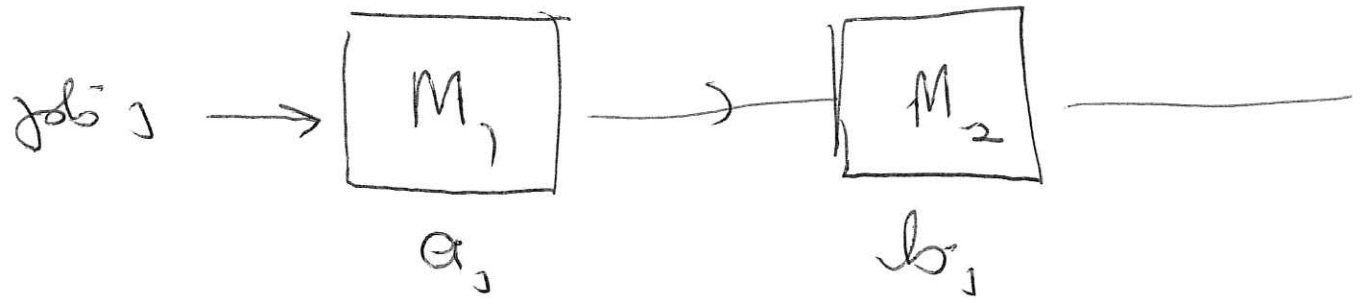


$$c_k^{new} < c_j^{old}$$
$$\& c_k^{new} = c_k^{old}$$



Two machine problem

$F_2 | - | C_{max}$



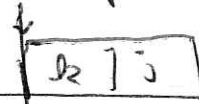
Flow shop: every job is processed by  $M_1$  and then  $M_2$ .

Permutation Flow Shop: same order on each machine

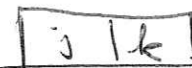
We can assume a permutation flow shop

$j$  precedes  $k$  on  $M_1$

jobs finished on  $M_1$



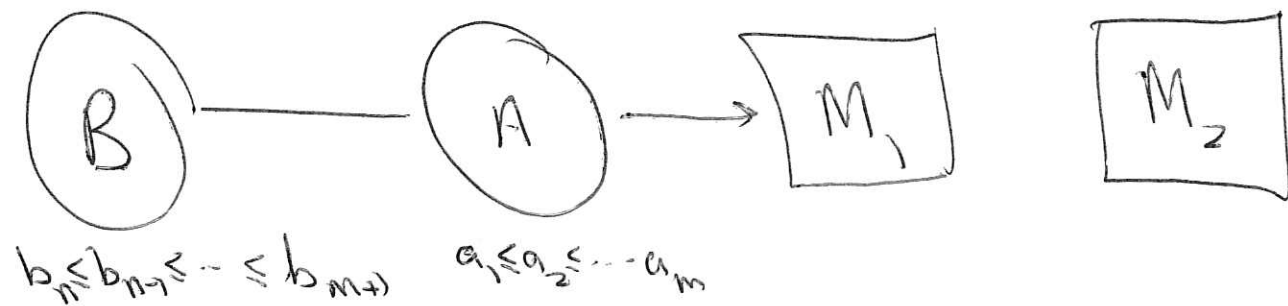
$C_{max}$  does not increase



# Johnson's Rule

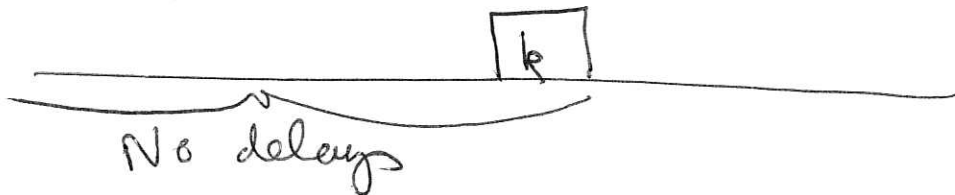
$$A = \{i : a_i \leq b_i\}$$

$$B = [n] \setminus A$$

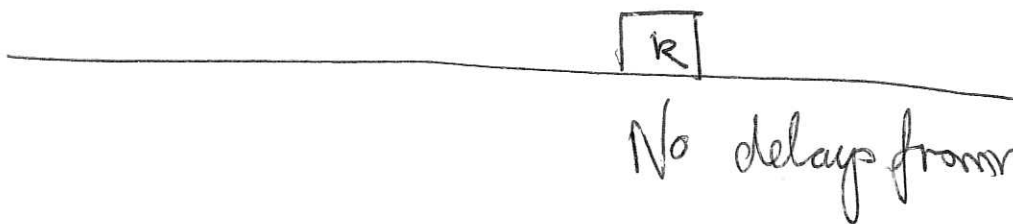


For all permutation schedules:

$M_1$



$M_2$



$C_{max} = \text{Sum of } n+1 \text{ job times. If you reduce each time by } \lambda, \text{ optimal order doesn't change.}$

$a_j =$  processing time on  $M_1$   
 $b_j =$  processing time on  $M_2$

$M_1$

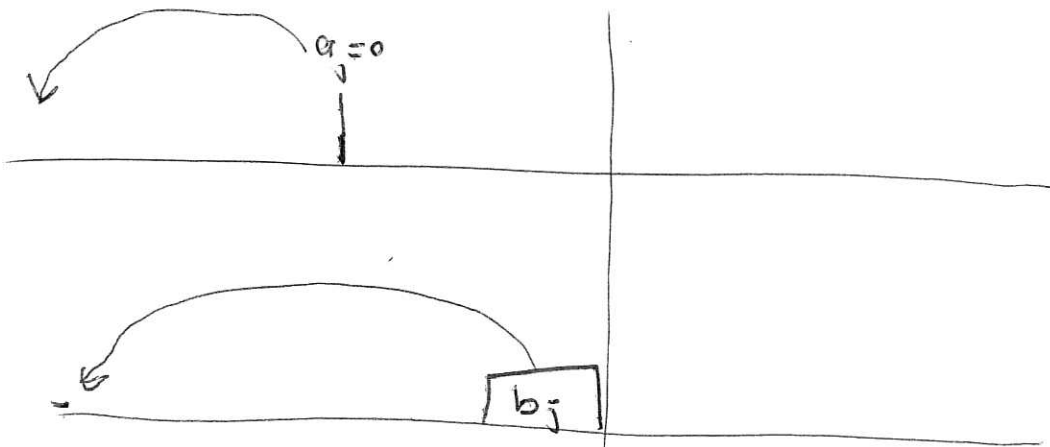


$M_2$



Reduce by  $\lambda$  until say  $a_j = 0$ .

Claim = put  $j$  first.



$b_j = 0$ , put it last.