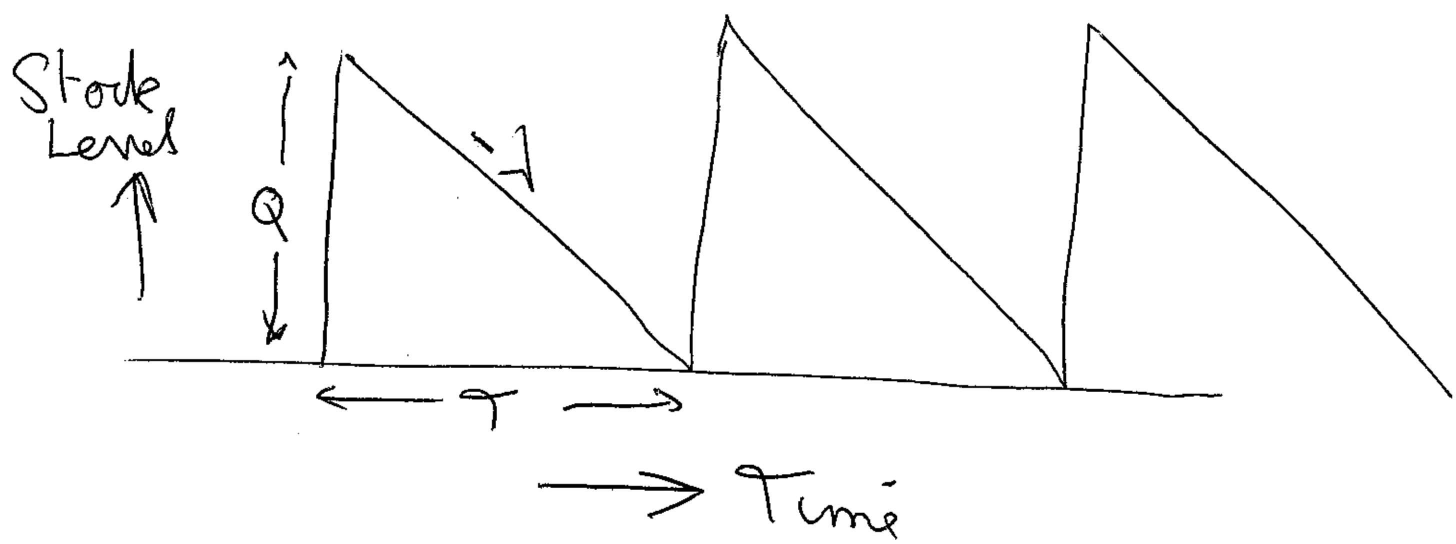
10\19\15

Stock or Inventory Control

Model 1

Demand for product is constant and λ per period. At constant time intervals of length T we make an order of Q items.

Problem: Optimize choice of Q, T.



A: fixed cost associated with making an order

C: cost per item

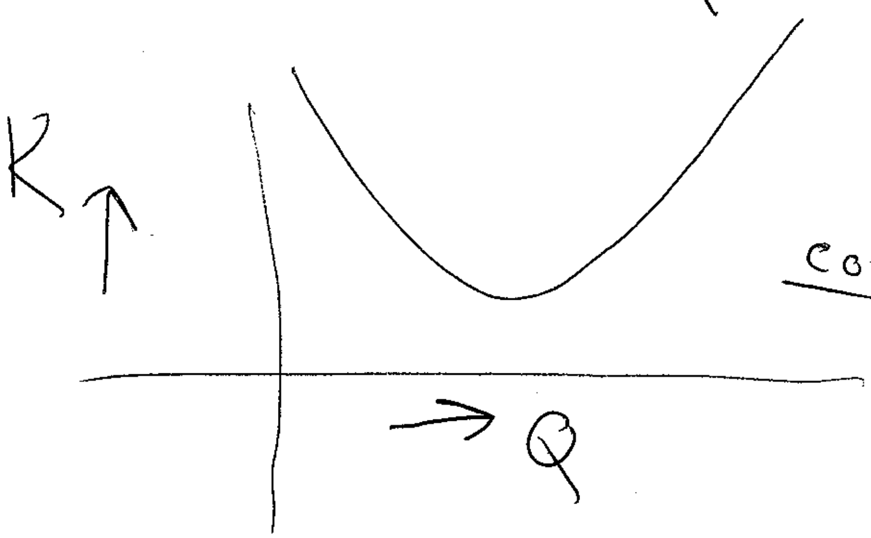
I: inventory carrying charge. One unit of stock kept for t periods cost It

Total cost per period = Order cost per period + Inventory cost per period

$$\frac{A}{T} + \frac{IQ}{2}$$

$$T = \frac{Q}{\lambda}$$

$$= \frac{A\lambda}{Q} + \frac{IQ}{2} \equiv K(Q)$$



convex

Set $\dfrac{dK}{dQ} = 0 \qquad -\dfrac{A\lambda}{Q^2} + \dfrac{I}{2} = 0$

Optimum: $Q_w = \sqrt{2\lambda A / I}$

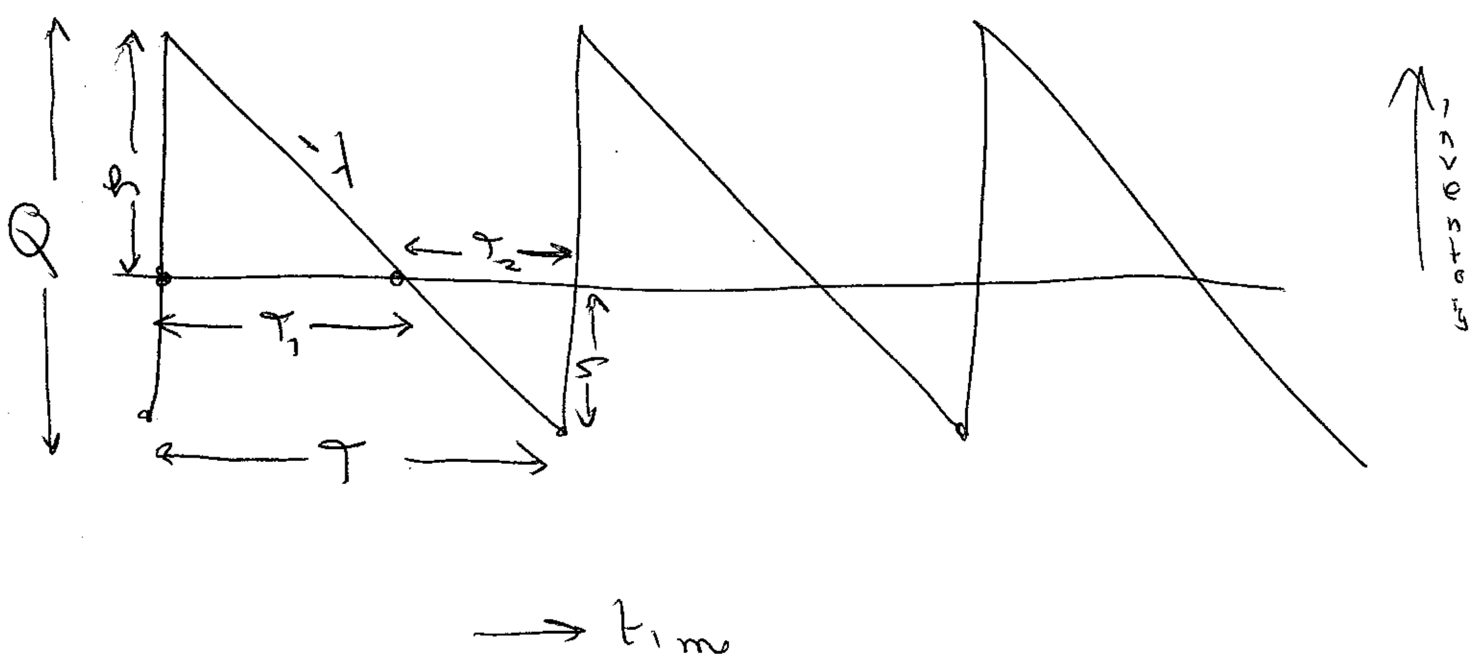
Wilson Lot-Size Formula.

$$T_w = \sqrt{2A / \lambda I}$$

$$K_w = \sqrt{2\lambda A I}$$

Non-convex

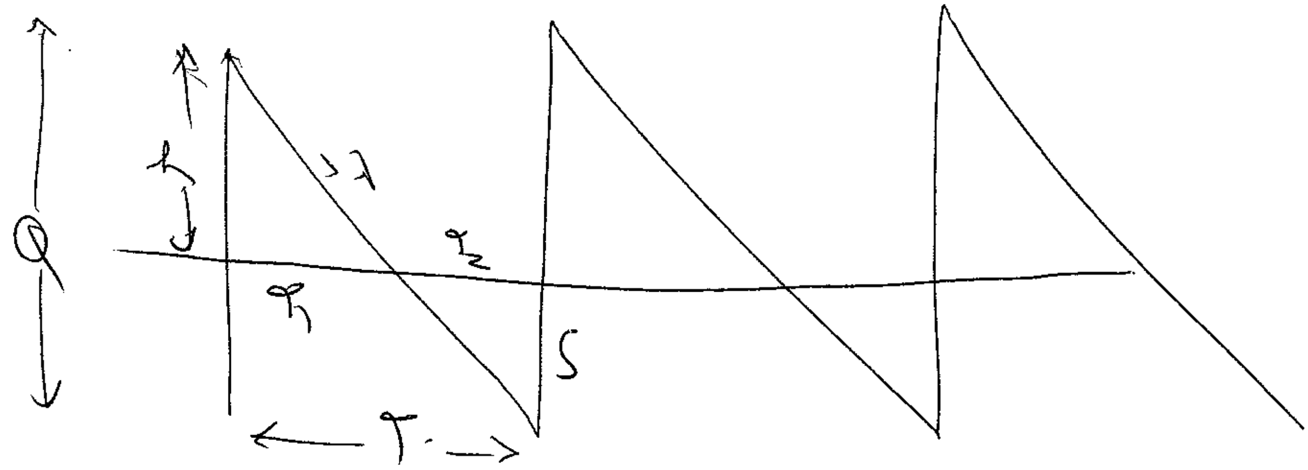
Model 2



We can go out of stock to back-order level of S.
We pay π penalty per per period per item out of stock.

Total cost

$$K = \underbrace{\frac{A\lambda}{Q}}_{\text{order cost}} + \underbrace{\frac{T_1}{T} \times \frac{h}{2} \times I}_{\text{inventory cost}} + \underbrace{\frac{T_2}{T} \times \frac{S}{2} \times \pi}_{\text{penalty cost}}$$



$$T_1 + T_2 = T \qquad K = \frac{A\lambda}{Q} + \frac{T_1 h I}{2T} + \frac{T_2 S \pi}{2T}$$

$$h + S = Q$$

$$h = \lambda T_1$$

$$S = \lambda T_2$$

$$Q = \lambda T$$

use these to write everything in terms of S & Q.

⋮

$$K = \frac{A\lambda}{Q} + \frac{I(Q-S)^2}{2Q} + \frac{\pi S^2}{2Q}$$

$$\frac{\partial K}{\partial Q} = \frac{\partial K}{\partial S} = 0$$

$$S^* = \left(\frac{2\lambda A I}{\pi(\pi+I)}\right)^{\frac{1}{2}} \qquad Q^* = Q_w\left(\frac{\pi+I}{\pi}\right)^{\frac{1}{2}}$$

$$K^* = K_w\left(\frac{\pi}{\pi+I}\right)^{\frac{1}{2}}$$

optimal

Model 3