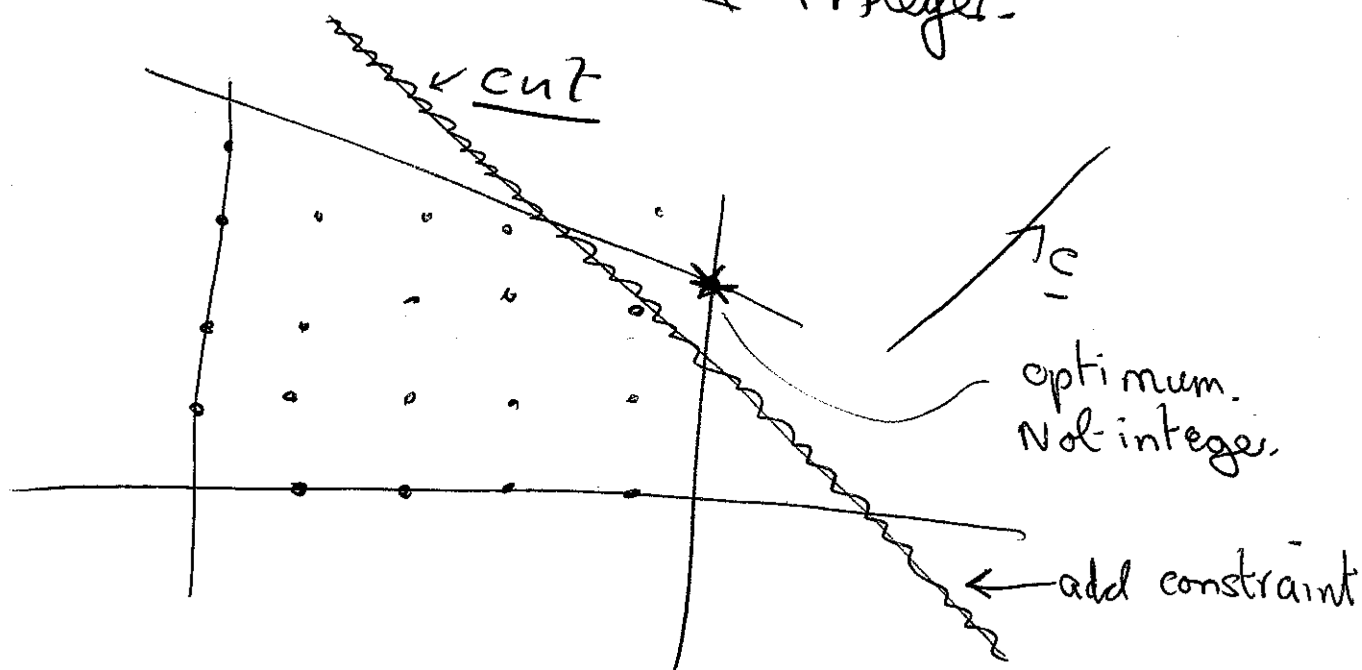
10/7/15

Integer Programming

Minimise $c.x$

$$Ax = b$$

$$x \geq 0 \text{ \& integer.}$$



Might try to solve LP (i.e. ignore integrality). If lucky, your answer is integer and you are done.

After adding cut  new set of feasible points is the same as the old set.

Gomory Cuts

Suppose we have solved the LP relaxation and there is a basic variable that is non-integer.

We have a row

$$x_i + \sum_{j \text{ non-basic}} b_{ij} x_j = b_{i0}$$

not an integer

Suppose we have a constraint

$$a_1 x_1 + a_2 x_2 + \cdots + a_n x_n = b \neq \text{integer}$$

and all x_i have to be integer and ≥ 0.

Write $a_j = \lfloor a_j \rfloor + f_j$

$b = \lfloor b \rfloor + f$

$6\frac{2}{3} = 6 + \frac{2}{3}$

$-6\frac{2}{3} = -7 + \frac{1}{3}$

Then

$$\underbrace{-\sum_{j=1}^{n} \lfloor a_j \rfloor x_j + \lfloor b \rfloor}_{\text{integer}} = -f + \underbrace{\sum_{j=1}^{n} f_j x_j}_{} \geq -f > -1$$

So $\sum_{j=1}^{n} f_j x_j \geq f$

Going back to

$$x_i + \sum_{j} b_{ij} x_j = b_{i0}$$

$$\sum_{j} f_j x_j \geq f > 0$$

This is not satisfied by $x_i = b_{i0}$ and $x_j = 0$, j non-basic

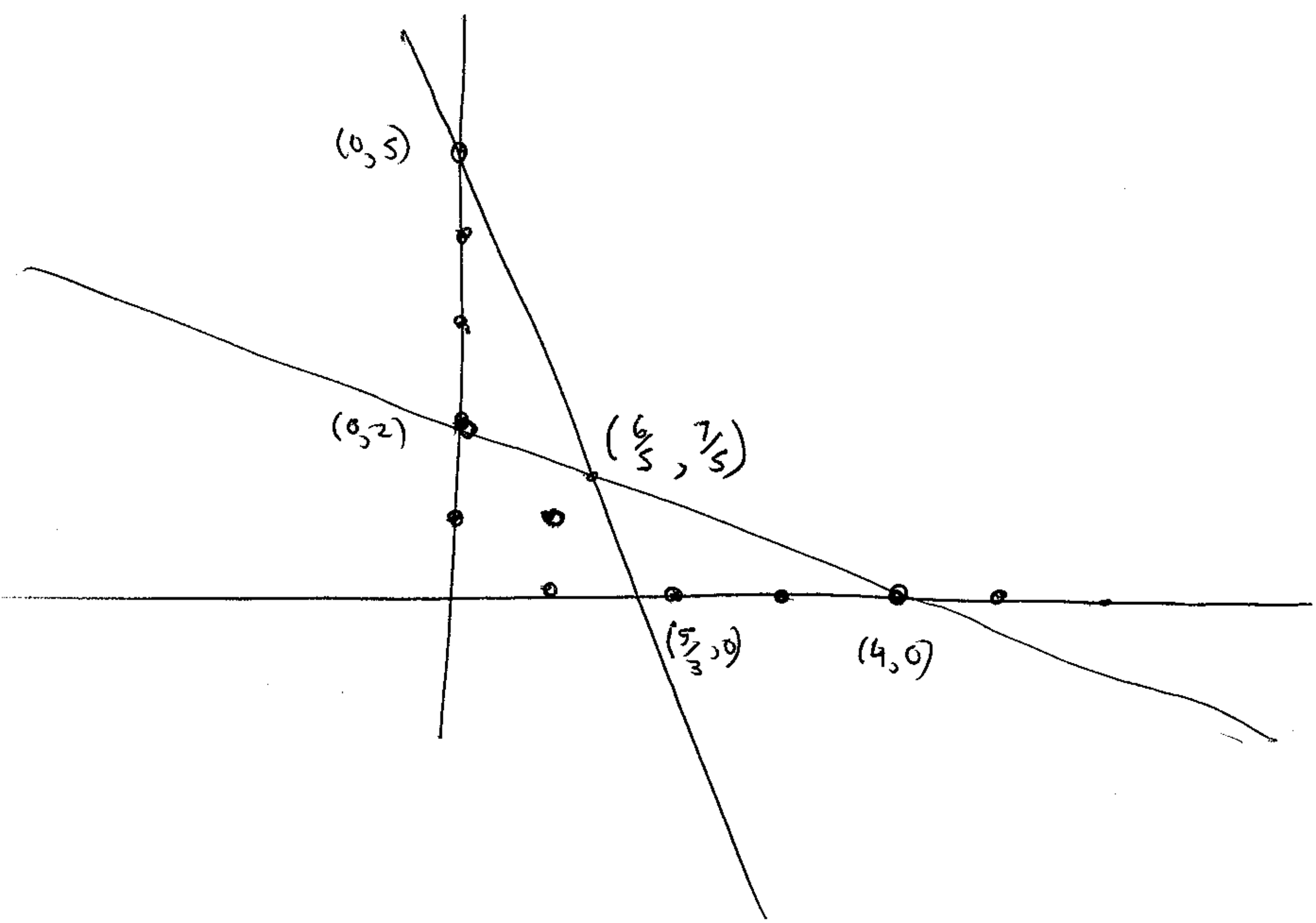
Example

$$\text{maximize} \quad 3x_1 + 2x_2$$

$$\text{subject to} \quad x_1 + 2x_2 \leq 4$$

$$3x_1 + x_2 = 5$$

$$x_1, x_2 \geq 0 \ \& \text{ integer}$$



BV	x_1	x_2	x_3	x_4	R
x_0	-3	-2			0
x_3	1	2	1		4
x_4	3	1		1	5
x_0		-1		1	5
x_3		$5/3$	1	$-\frac{1}{3}$	$7/3$
x_1	1	$\frac{1}{3}$		$\frac{1}{3}$	$5/3$
x_0			$3/5$	$4/5$	$32/5$
x_2		1	$3/5$	$-1/5$	$7/5$
x_1	1		$-\frac{1}{5}$	$2/5$	$6/5$

$x_0: \; x_0 + \frac{3}{5}x_3 + \frac{4}{5}x_4 = \frac{32}{5}$

$\frac{3}{5}x_3 + \frac{4}{5}x_4 \geq \frac{2}{5}$

BV	x_1	x_2	x_3	x_4	S_1	S_2	
x_0			$3/5$	$4/5$			$32/5$
x_2		1	$3/5$	$-1/5$			$7/5$
x_1	1		$-1/5$	$2/5$			$6/5$
S_1			$-3/5$	$-4/5$	1		$-2/5$

x_0					1		6
x_2		1		-1	1		1
x_1	1			$2/3$	$-1/3$		$4/3$
x_3			1	$4/3$	$-5/3$		$2/3$

Cut: $\frac{2}{3}x_4 + \frac{2}{3}S_1 \geq \frac{1}{3}$

x_0					1		6
x_2		1		-1	1		1
x_1	1			$2/3$	$-1/3$		$4/3$
x_3			1	$4/3$	$-5/3$		$2/3$
S_2				$-2/3$	$-2/3$	1	$-1/3$

BV	X_1	X_2	X_3	X_4	S_1	S_2	R
X_0					1		6
x_2		1			2	$-3/2$	$3/2$
x_1	1				-1	1	1
x_3			1		-3	2	0
X_4				1	1	$-3/2$	$1/2$
Cut						$\frac{1}{2}S_2 \geq$	$1/2$

BV	X_1	X_2	X_3	X_4	S_1	S_2	S_3	R
X_0					1			6
X_2		1			2	$-3/2$		$3/2$
X_1	1				-1	1		1
X_3			1		-3	2		0
X_4				1	1	$-3/2$		$1/2$
S_3						$-1/2$	1	$-1/2$

BV	X_1	X_2	X_3	X_4	S_1	S_2	S_3	R
X_0					1			6
X_2		1			2			3
X_1	1				-1			0
X_3			1		-3		4	-2
X_4				1	1		-3	2
S_2						1	-2	1

and so on