10/5/15

Integer Linear Programming

A company has to consider n projects for funding.

Project j is expected to earn c_j in profit but requires a_{ij} dollars in period $i = 1, 2, \ldots, m$.

There is b_i available for investment in period i.

Company want to maximise profit.

Let $x_j = \begin{cases} 1 & : \text{do project } j \\ 0 & : \text{do not do project } j \end{cases}$

$$\text{maximise profit} = c_1x_1 + c_2x_2 + \cdots + c_nx_n$$

subject to

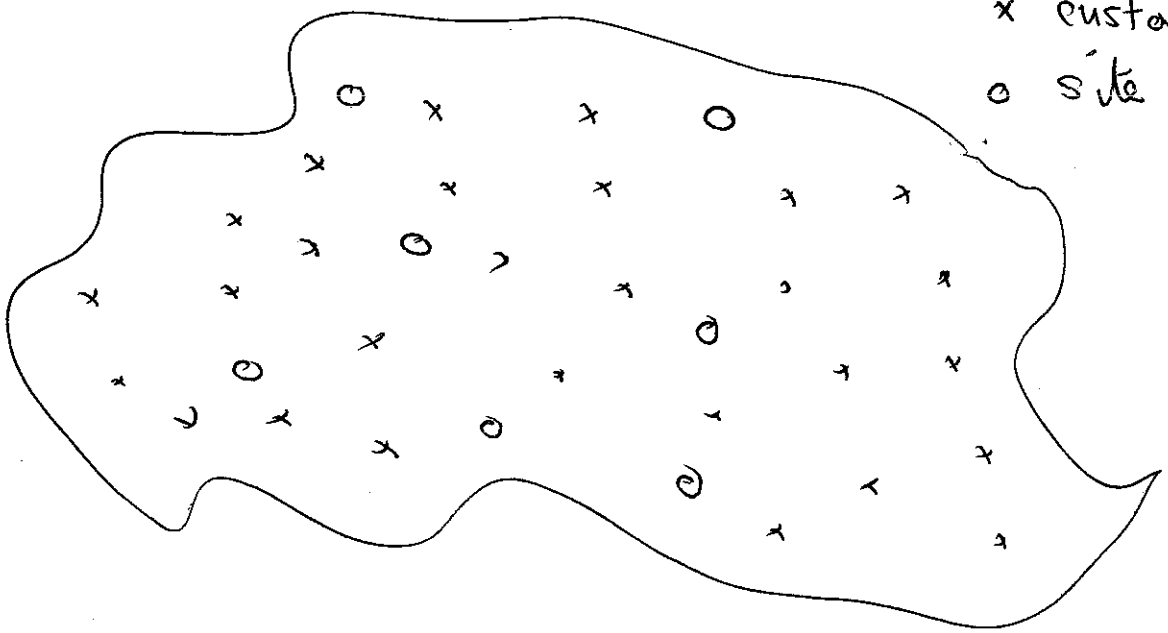
$$a_{i1}x_1 + a_{i2}x_2 + \cdots + a_{in}x_n \leq b_i \quad i = 1, 3, \ldots, m$$

$$0 \leq x_j \leq 1$$

x_j is an integer

Simple Plant Location Problem

A company has selected m possible site for depots to supply its customers.



f_i = cost of building depot i

c_{ij} = cost of supplying customer j from site i.

minimize $\sum_{i=1}^{m} f_i y_i + \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} x_{ij}$

construction cost + shipping cost

$y_i = \begin{cases} 1 : \text{depot at } i \\ 0 : \text{no depot at } i \end{cases}$

$x_{ij} = \begin{cases} 1 = i \text{ supplies } j \\ 0 : \neg\, i \text{ supplies } j \end{cases}$

$$\sum_{i=1}^{m} x_{ij} = 1 \qquad \forall j$$

$$0 \le x_{ij} \le y_i \qquad \forall i$$

y_i is an integer

(x_{ij} is an integer)

Suppose now there is a maximum capacity M_i for a depot at i and customer j requires R_j items.

Add the constraint:

$$\sum_{j=1}^{n} R_j X_{ij} \leq M_i \quad \forall i$$

Set Covering

$S_1, S_2, \ldots, S_m \subseteq [n]$

S_j costs c_j

I want to buy at least one/each item in $[n]$.

as cheaply as possible.

$$X_j = \begin{cases} 1 : \text{buy } S_j \\ 0 : \text{don't buy } S_j \end{cases}$$

minimize $c_1 x_1 + c_2 x_2 + \cdots + c_m x_m$

$a_{11} x_1 + a_{12} x_2 + \cdots + a_{1m} x_m \geq 1$

$\vdots$

$a_{n1} x_1 + a_{n2} x_2 + \cdots + a_{nm} x_m \geq 1$

$0 \leq x_j \leq 1$ & integer

Notation $a_{ij} = 1_{i \in S_j}$