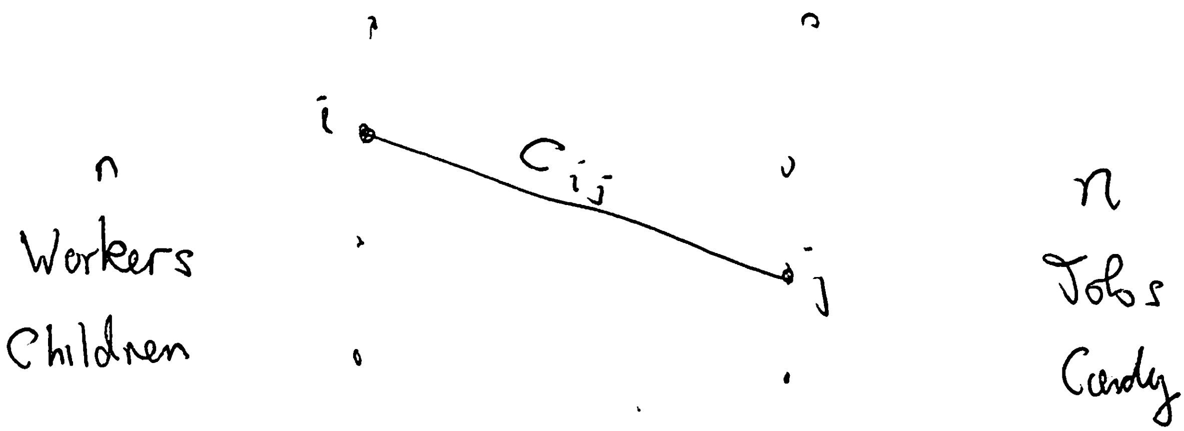
10\2\15

Assignment Problem



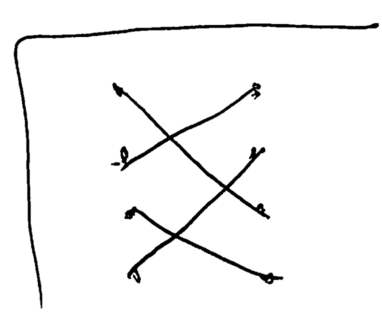
c_{ij} = cost of worker does job j

Problem: assign $i \longrightarrow \pi(i)$ to

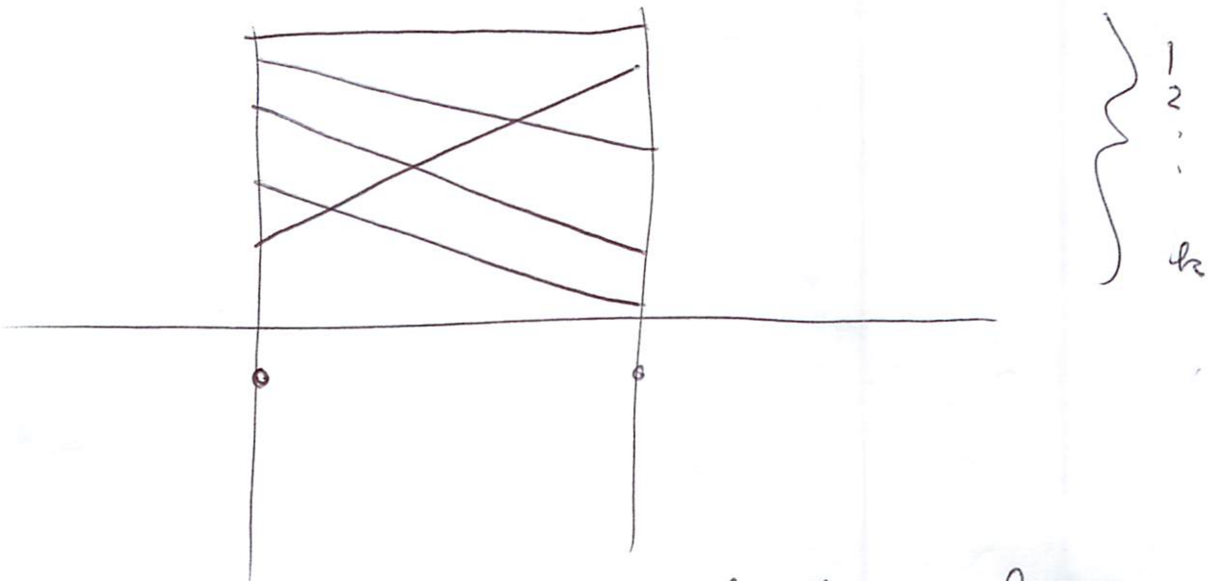
minimise $\sum_{i=1}^{n} c_{i\,\pi(i)}$

[In graph theory terms we see a minimum weight perfect matching]

Reduce solution to a sequence of shortest path problems

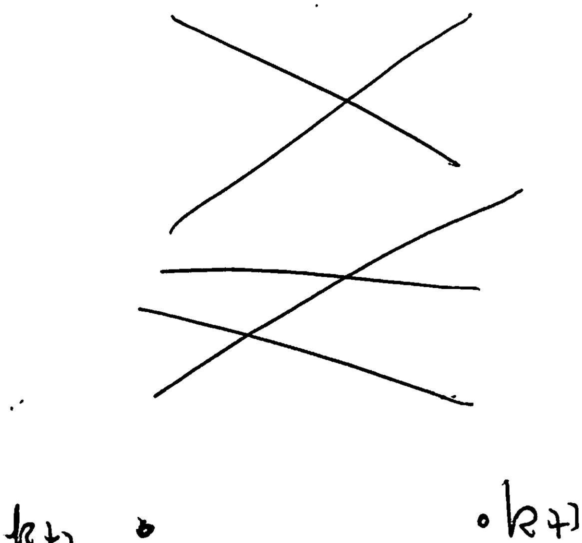


Suppose we have solved the $k \times k$ problem:

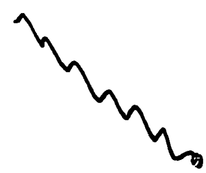


Given solution to $k \times k$ problem, solving $(k+1) \times (k+1)$ problem is a shortest path problem.

Suppose solution to $k \times k$ problem is a matching M



M' is any $(k+1)$-matching

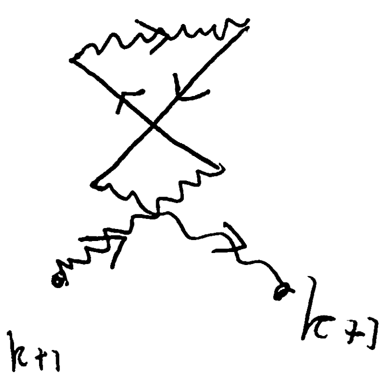


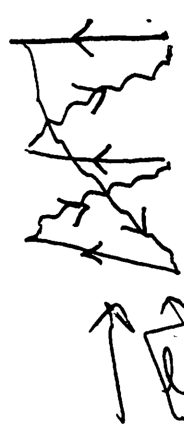
Choose M' to minimise

$$M' \oplus M = (M'/M) \cup (M/M')$$

$\text{cost}(M') - \text{cost}(M)$

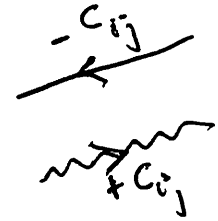
$= \sum \ell(\sim) - \sum \ell(/)$





$\boxed{\ell(C) \geq 0}$

Orient:



One of these

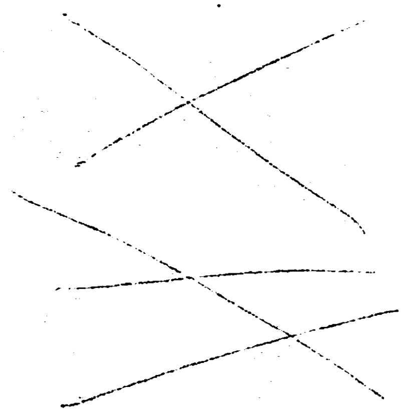
Several of these

Sum of lengths of paths + cycles =

If $\ell(C) < 0$ then it would give a better solution to $k \times k$ problem.

Suppose Solution is $k \times k$ problem to a matching M.

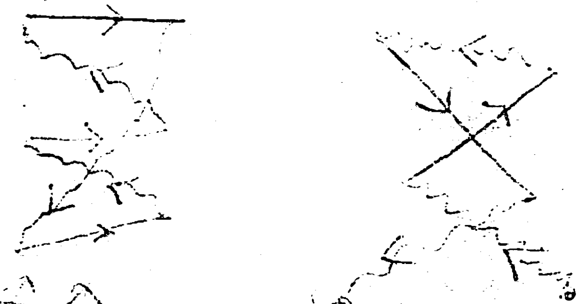
M' is any $(k+1)$-matching



$M \oplus M' = (M \setminus M') \cup (M' \setminus M)$

$\text{cost}(M') - \text{cost}(M) \geq$

$= \sum c(e) - \sum c(e)$



$\boxed{c(C) \geq 0}$

Sum (weight) of paths + cycles =

Some of these

One of these

If $c(C)<0$ then it would give a better solution to the k-problem

Integer Programming

Linear programming where you have to keep some variables integer. —— Hard in general.

Assignment Problem:

Special I.P.

If I replace ⊛ by $0 \le x_{ij} \le 1$ and use the Simplex algorithm then ⊛ holds in optimum

$$\text{minimize} \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij}$$

$$\text{subject to} \quad \sum_{i=1}^{n} x_{ij} = 1, \quad \forall j$$

$$\sum_{j=1}^{n} x_{ij} = 1, \quad \forall i$$

$$x_{ij} = 0 \text{ or } 1 \qquad \circledast$$