Production Problem

Demand is not known exactly
Probability demand in period $n$ in $d = p_{n,d}$

Minimize expected cost

$$f_n(h) = \min_x \left[ C(x) + \sum_{d=0}^{\infty} p_{n,d} \left\{ \Pi(d-x-h)^+ + f_{n+1}(h+x-d)^+ \right\} \right]$$

current stock level

Assume demand is not known when we decide production level.
Minimize expected cost

\[ f_n^*(h) = \sum_{d=0}^{\infty} \min \left\{ C(x) + \Pi (d-x-h)^+, \ f_{n+1}((h+x-d)^+) \right\} \]

Assume demand is known when we decide production level.

\[ f_n^*(h) \leq f_n^*(h) \]

\[ \mathbb{E}(\min_x) \leq \min_x \mathbb{E}(\ ) \]
Alternatively we can produce enough to meet demand with >90% probability

\[ f_n(h) = \min_x \left\{ C(x) + \sum_{d=0}^{\infty} P_{n,d} \left( \frac{\Pi(d-x-h)^+}{f_{n+1}(h+x-d)} \right) \right\} \]

where

\[ \alpha_h = \min \{ \alpha : \sum_{d=\alpha+h}^{\infty} P_{n,d} \leq .1 \} \]
Minimise time to arrive at \( F_i \)

\[ D_{i,j} = \text{set of possible decisions.} \]

\[ f_r(i) = \min_{d \in D_{i,j}} \left[ \sum_{t, j} \rho_{d}(r, i, j, t)[t + f_{r+1}(i)] \right] \]

Minimum expected time to reach \( F_i \) from \( (r, i) \)