\[ \text{maximize } \gamma \]
\[ \text{subject to } \sum_{i=1}^{n} a_{ij} q_i \leq b_i, j = 1, \ldots, m \]
\[ q_1 + \ldots + q_m = 1 \]
\[ q_i \geq 0 \]

\[ \text{minimize } \sum_{i=1}^{n} a_{ij} p_i \leq 0, j = 1, \ldots, n \]
\[ p_1 + \ldots + p_n \leq 1 \]
\[ p_i \geq 0 \]

\[ \text{maximize } c_1 x_1 + c_2 x_2 + c_3 x_3 \]
\[ \text{subject to } a_{11} x_1 + a_{12} x_2 + a_{13} x_3 \leq b_1 \]
\[ a_{21} x_1 + a_{22} x_2 + a_{23} x_3 \geq b_2 \]
\[ a_{31} x_1 + a_{32} x_2 + a_{33} x_3 = b_3 \]
\[ x_1 \geq 0, x_2 \leq 0 \]
Dominant Strategies

\[ \begin{pmatrix} 6 & 5 & 3 & 4 \\ 2 & 2 & 2 & 3 \\ 1 & 2 & 2 & 3 \end{pmatrix} \]

Row 1 dominates Row 2

Col 3 dominates rest
Simple Games

(i) Symmetric Game

\[ A^T = -A \]

\[ \begin{bmatrix} 0 & 2 & -1 \\ -2 & 0 & 3 \\ 1 & -3 & 0 \end{bmatrix} \]

\[ \forall \mathbf{p}: \text{PAY}(\mathbf{p}, \mathbf{p}) = 0 \]

\[ \text{PAY} \begin{bmatrix} \end{bmatrix} \]

\[ \sum_{i \leq j} a_{i,j} p_i p_j = 0 \]

\[ a_{i,j} p_i + a_{j,i} p_j = 0 \]

\[ p_A \leq 0 \]

\[ p_B \geq 0 \]
General Games

$n$ players.

Player $i$ has $m_i$ pure strategies.

$A_k(i_1, i_2, \ldots, i_n) = \text{pay-off to player } k$

of player $i$ play $i_t$, $t = 1, 2, \ldots, n$. 

Mixed Strategies: player $i$ has a vector of probabilities.

$\Delta_k(p_1, p_2, \ldots, p_n)$
$p_1^*, p_2^*, \ldots, p_n^*$ is a Nash equilibrium if

$$A_k(p_1^*, p_2^*, \ldots, p_k^*, \ldots, p_n^*)$$

$$\geq A_k(p_1^*, p_2^*, \ldots, p, \ldots, p_n^*)$$

$$\forall k \in P$$

Thm: There is always at least one Nash equilibrium (of mixed strategies). Hard to compute a Nash Equilibrium.
1 unit of "traffic flow" from $s \rightarrow t$

$N = 10^{10}$ cars.

Nash Eq: Everyone along bottom path $N$ with $\alpha : N\left(\alpha + (1-\alpha)^2\right) = \frac{3}{4}N$

$= N\left(1 - \alpha + \alpha^2\right)$ minimized at $\alpha = \frac{1}{2}$