Two Person Zero Sum Games.

Player A  \[ \begin{bmatrix} i & \text{matrix A} & \text{Player B} \\ x & \end{bmatrix} \]

\[ A[i,j] = xc \]

Player B pays \( xc \) units to Player A.

\( A = \) Row Player: Chooses \( i \)

\( B = \) Column Player: Chooses \( j \)
\[
\begin{bmatrix}
5 & 0 \\
0 & 1
\end{bmatrix}
\]

Tennis

A

Serving

B

Soccer
A game is played over and over again.
A tries to maximize average winnings.
B tries to minimize A's average winnings.

\( S_A : \{ \text{A's possible strategies} \} \)

\( S_B \)

\( (i) = i, i, i, i, i, \ldots \)

**Pure Strategy**
If $u \in S_A$
$\forall v \in S_B$
then $PAY(u, v) = \text{average payoff to } A$

**Stable Solution:**

$(u_0, v_0)$ is stable if

$PAY(u_0, v) \geq PAY(u_0, v_0) \geq PAY(u, v_0)$

$u_0$ is the largest entry in column $v_0$.
$v_0$ is the smallest entry in row $u_0$. 

$(u_0, v_0)$
\[
P = \begin{bmatrix} 0 & 1 & -1 \end{bmatrix} \\
R = \begin{bmatrix} -1 & 0 & 1 \end{bmatrix} \\
S = \begin{bmatrix} 1 & -1 & 0 \end{bmatrix}
\]

Is there a stable solution using pure strategies?

Is there \((u_0, v_0)\) which is a column max and row min?

Every row min = -1
Every col max = 1
What are A's guaranteed winning

\[ P_A = \max_u \text{ROWMIN}(w) \]

If A chooses u then in long run A will get
\[ \text{ROWMIN}(w) \]

\[ u \begin{bmatrix} \star \star \star \star \end{bmatrix} \]
$B$ should lose no more than

$$P_B = \min_{\nu} \text{COLMAX}(\nu)$$

Claim:

1. $P_A \leq P_B$
2. $P_A = P_B$ iff $\exists$ stable solution
Claim:

(i) \( P_A \leq P_B \)

(ii) \( P_A = P_B \) iff \( \exists \) stable solution

\[
\begin{array}{c}
S_A \\
\hline
u \\
\hline
v \\
\hline
x \\
\hline
S_B
\end{array}
\]

ROWMIN \((u)\) \leq x \leq COLMAX \((u)\)

largest

smallest