Department of Mathematical Sciences
Carnegie Mellon University

21-484 Graph Theory

Test 2

Name:____________________________

Problem	Points	Score
1	33	
2	33	
3	34	
Total	100	

Q1 (34 points)
A graph $G = (V, E)$ is *Hamilton connected* if for every pair of distinct vertices $v_1, v_2 \in V$ there is a Hamilton path with endpoints v_1, v_2. Show that if G is Hamilton connected then $\delta(G) \geq 3$.

Q2 (33 points)
Let G be a simple 3-regular graph. In this question you are asked to show that $\kappa'(G) = \kappa(G)$. Draw pictures to illustrate the proof.
(a) Show that if G has a cut-point then G has a cut-edge.
(b) Show that if $G - \{u, v\}$ has ≥ 2 components $C_1, C_2, \ldots$ then G can be disconnected by deleting 2 edges incident with u or v.
Deduce that $\kappa'(G) = \kappa(G)$.

Q3 (33 points)
A matching M of graph $G = (V, E)$ is *maximal* if $M + e$ is not a matching for any $e \in E \setminus M$. Let U be the set of M-unsaturated vertices. Why is U an independent set?
Let M^* be a matching of maximum size in G. Show that every path/cycle in $M \Delta M^*$ contains at least 1/2 as many M edges as M^* edges. Why might this not be the case if M were not maximal?
Dduce that $|M| \geq |M^*|/2$.