Department of Mathematics
Carnegie Mellon University

21-228 Discrete Mathematics
Test 4

Name:____________________________________

Problem	Points	Score
1	33	
2	33	
3	34	
Total	100	

Q1: (33pts)
The simple graph G has n vertices and maximum degree $n-2$ and diameter 2. Prove that it has at least $2n-4$ edges. [Diameter 2 means that there is a path of length 2 joining any 2 non-adjacent vertices.]
[Hint: Take a vertex x of degree $n-2$ and let y be the unique non-neighbour of x. Let N be the set of neighbours of x and M be the set of neighbours of y. Prove that each $v \in N \setminus M$ has a neighbour in M. Now count the number of edges you know exist.]

Q2: (33pts)
Let G be a bipartite graph with vertex partition A, B where $|A| = |B| = n$. Suppose that each vertex of A has degree at least $3n/4$ and each vertex of B has degree at least $n/4$. Prove that G has a perfect matching.

Q3: (34pts)
A tree has n vertices all of which have degree 1 or d for some integer $d \geq 2$. Suppose n_1 are of degree 1 and n_d are of degree d. Prove that

$$n_d = \frac{n-2}{d-1}.$$

(Hint: write down 2 linear equations involving n_1 and n_d or use induction.)