1. Suppose that $A_1, A_2, \ldots, A_n \subseteq A$ and $|A_i| = k$ for $i = 1, 2, \ldots, n$ and that $q$ is a positive integer. Show that if $nq \left(1 - \frac{1}{q}\right)^k < 1$ then the elements of $A$ can be $q$-colored so that each $A_i$ contains an element of each color.

2. Let $G = (V, E)$ be a graph on $n$ vertices, with minimum degree $\delta > 1$. Show that $G$ contains a dominating set of size at most $n \frac{\log(\delta+1)}{\delta+1}$.

3. Prove that there is an absolute constant $c > 0$ with the following property. Let $A$ be an $n \times n$ matrix with pairwise distinct real entries. Then there is a permutation of the rows of $A$ so that no column in the permuted matrix contains an increasing subsequence of length at least $c \sqrt{n}$.

The following inequalities might be useful:

\[
\binom{n}{k} \leq \left(\frac{ne}{k}\right)^k \quad \text{and} \quad 1 + x \leq e^x \quad \text{and} \quad n! \geq \left(\frac{n}{e}\right)^n.
\]