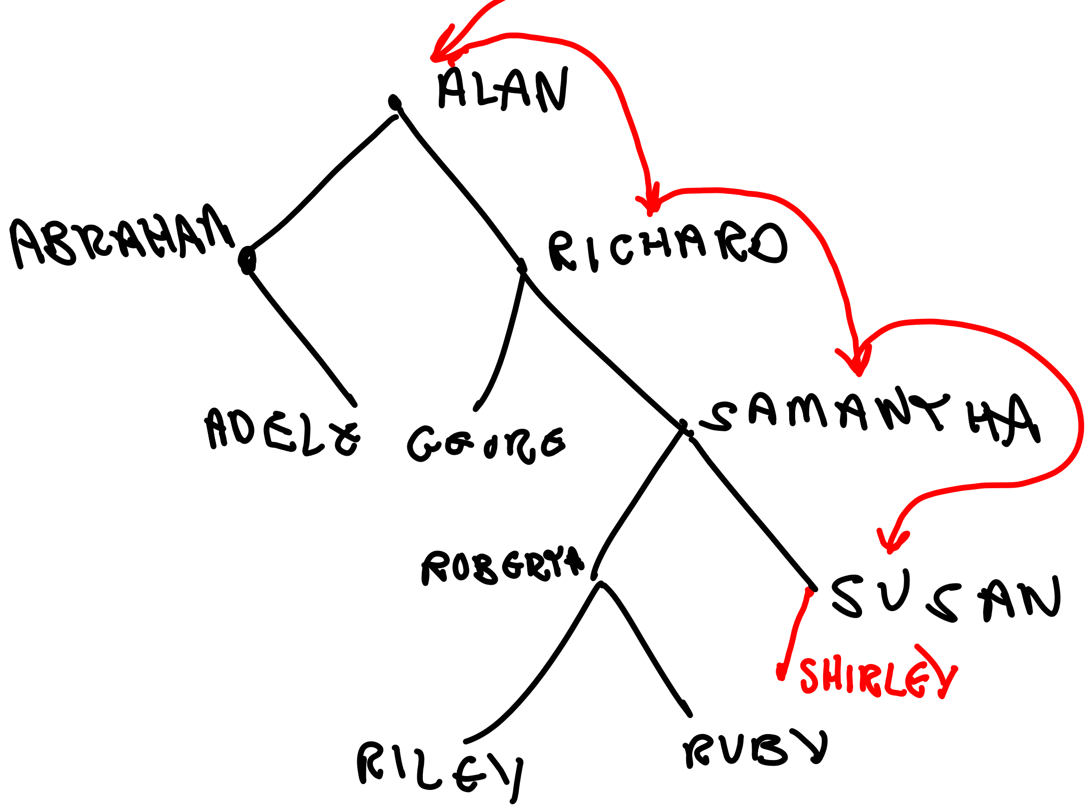
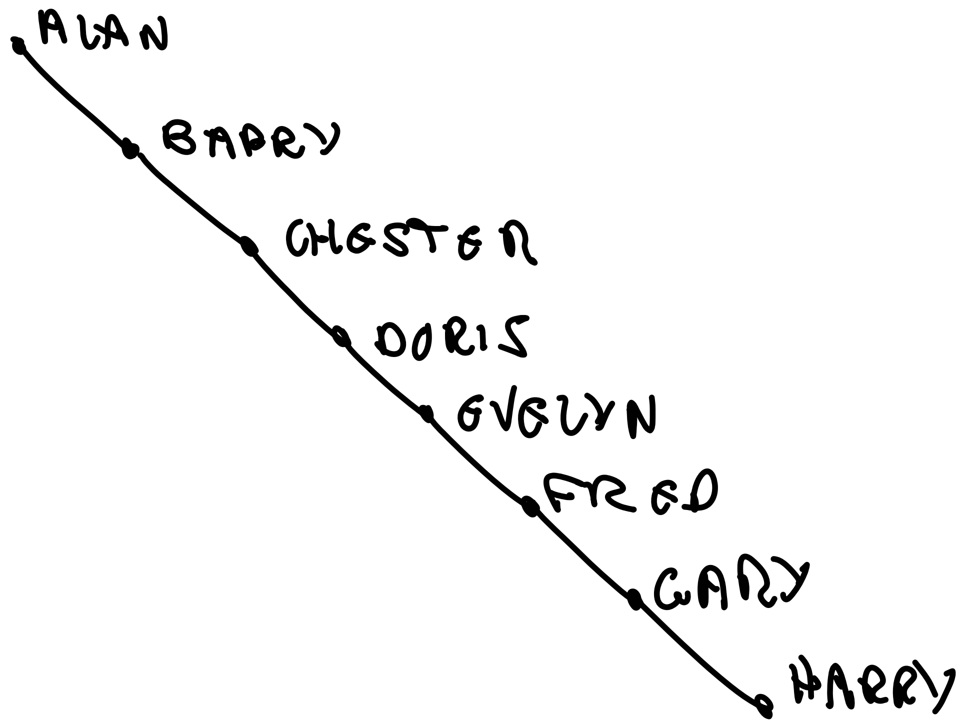
9/27/10



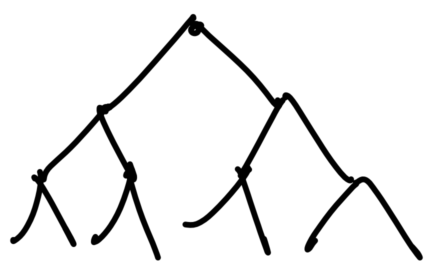
Time to find or insert an item is proportional (at worst) to depth of the tree

BAD TREE



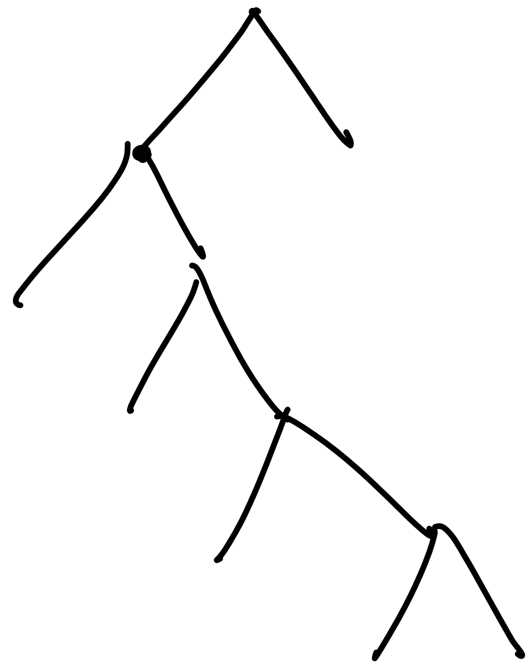
Is IVAN in "club"

GOOD TREE

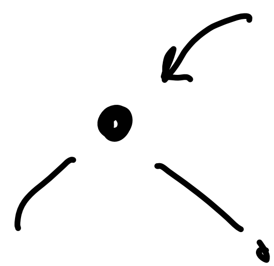


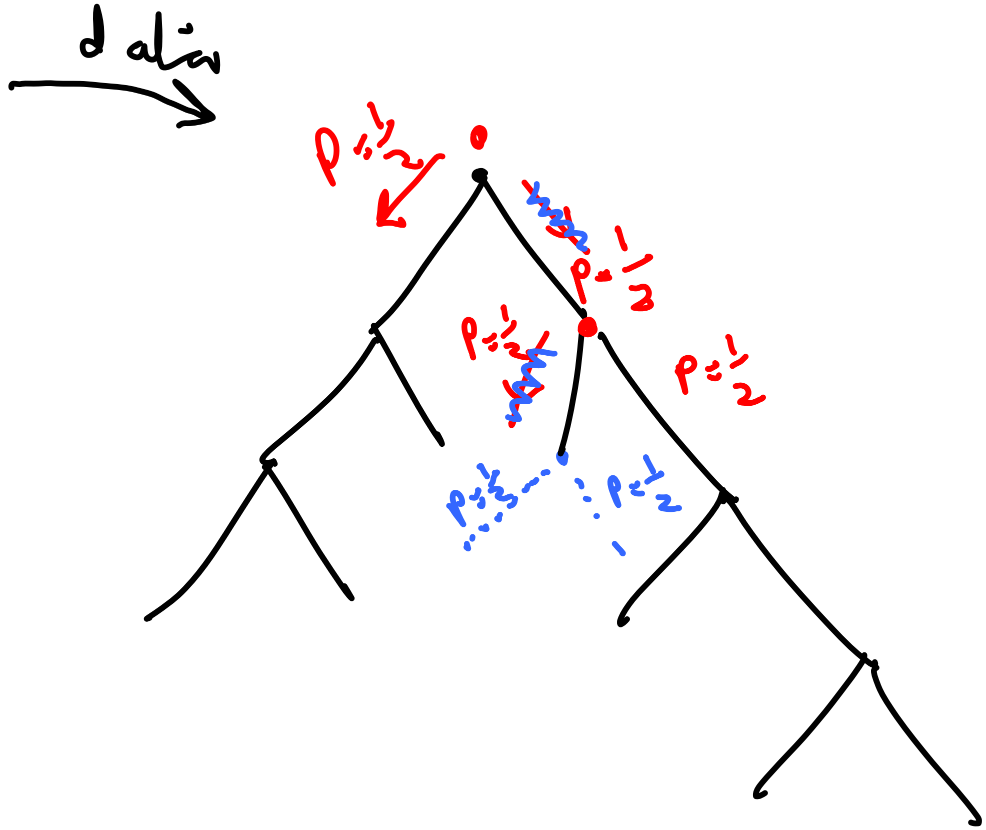
n items

Depth $\sim \log_2 n$



Rule for insertion.





T_n

What is the likely depth after adding n items.

D_n = depth of the tree T_n.

Claim:

$$P[D_n \geq t] \leq \left(n\, 2^{-(t-1)/2}\right)^t$$

What then is the chance that $D_n \geq 3\log_2 n$?

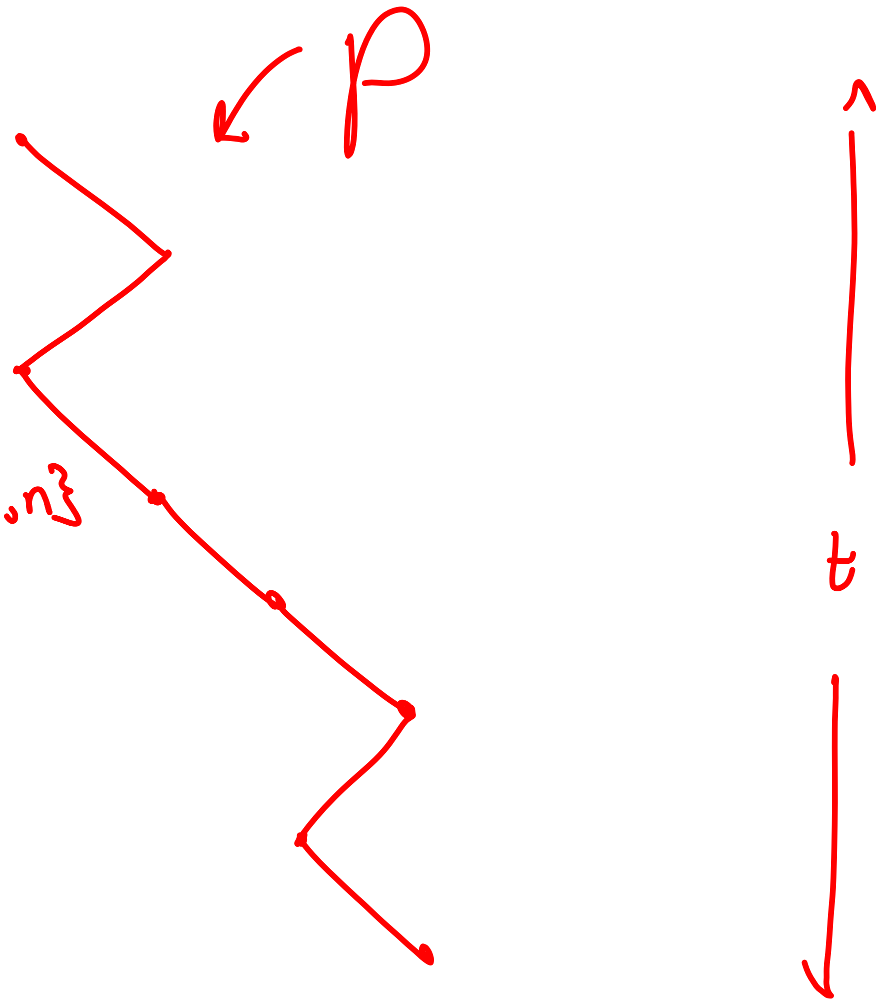
$$P(\ldots) \leq \left[n\, 2^{-(3\log_2 n - 1)/2}\right]^{3\log_2 n}$$

$$= \left[n \times 2^{\frac{1}{2}} \times n^{-3/2}\right]^{3\log_2 n} \ll 1$$

$$\mathrm{DEEP} = \{D_n \geq t\} = \bigcup_{P,S} \mathrm{DEEP}(P,S)$$

$\Downarrow$

$\exists$

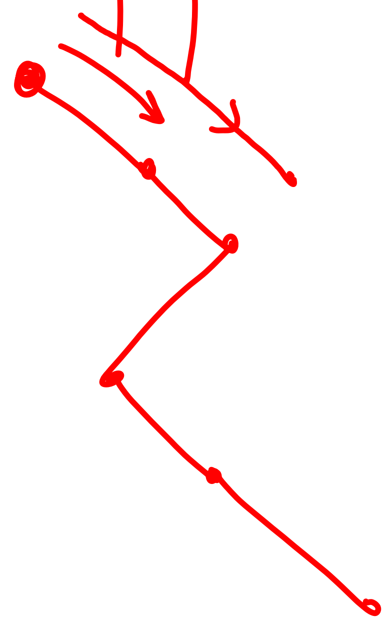


& $S \subseteq \{1,2,\dots,n\}$

{particles that make P}

$$\Pr[\mathrm{DEEP}] \leq \sum_{p} \sum_{S} \Pr[\mathrm{DEEP}(p, S)]$$

$$\leq \sum_{p} \sum_{S} \left(\frac{1}{2}\right)^{1+2+3+\cdots+t}$$



$$= \sum_{p} \sum_{S} \left(\frac{1}{2}\right)^{t(t+1)/2}$$

2^t of these

$\binom{n}{t}$ of these

$$\leq \binom{n}{t} 2^t \left(\frac{1}{2}\right)^{t(t+1)/2}$$

$< n^t$

Random Variables

A function $\zeta : \Omega \to \mathbb{R}$

zeta

is a random variable

Two Dice (x_1, x_2)

$$\zeta(x_1, x_2) = x_1 + x_2$$

$p_k = P_r(\zeta = k)$

k	2	3	.....
p_k	$\frac{1}{36}$	$\frac{1}{18}$	

Colored Balls

$\Omega = \{$ m indistinguishable balls,
n colors $\}$

Uniform Distribution

ζ = # of colors used

choose colors

choices of colorings so that each color is used.

$$P_k = \frac{\binom{n}{k}\binom{m-1}{k-1}}{\binom{n+m-1}{n-1}} \leftarrow |\Omega|$$

Binomial Random Variable $B_{n,p}$

n coin tosses

$p = \Pr(\text{Heads})$

$\xi = \#$ of heads

$\sum_{\omega} P(\omega)$

$\omega = HTTHT\ldots T$

k heads

$$\Pr(B_{n,p} = k) = \binom{n}{k} p^k (1-p)^{n-k}$$

coins which come down heads

Expectation = "Averge"

ζ is a random variable

$$E(\zeta) = \sum_{\omega \in \Omega} \zeta(\omega) P(\omega)$$

$$= \sum_{k} k \, P_r(\zeta = k)$$

2 dice

$$E(\zeta) = 2 \times \frac{1}{36} + 3 \times \frac{2}{36} + \cdots + 12 \times \frac{1}{36} = 7$$

n colors; m balls

ζ = # colors used

$$E(\zeta) = \sum_{k=1}^{n} k \frac{\binom{n}{k}\binom{m-1}{k-1}}{\binom{n+m-1}{n-1}}$$

$$k\binom{n}{k} = n\binom{n-1}{k-1}$$

$$=$$

$$= n \sum_{k=1}^{n} \frac{\binom{n-1}{k-1}\binom{m-1}{m-k}}{\binom{n+m-1}{n-1}}$$

Vandermonde

$$\vdots$$

$$\frac{mn}{n+m-1}$$