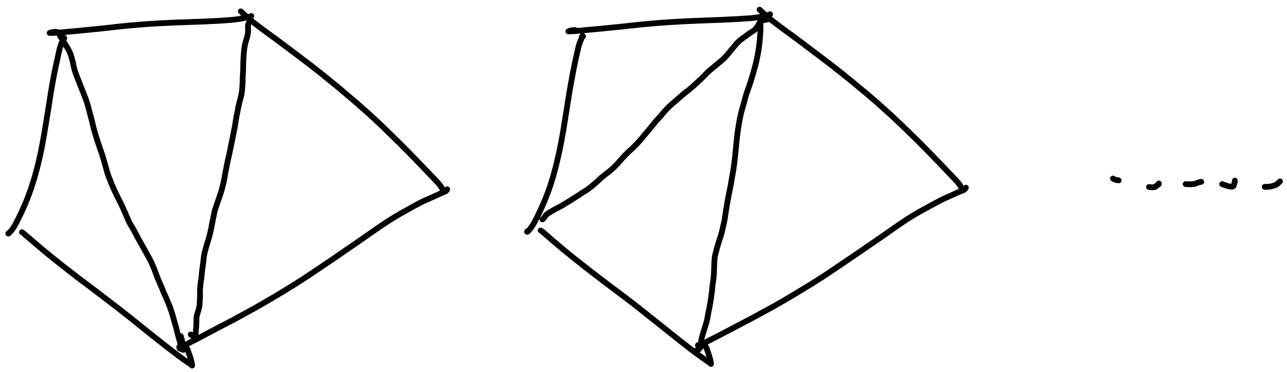
9\13\10

# of triangulations of a polygon.



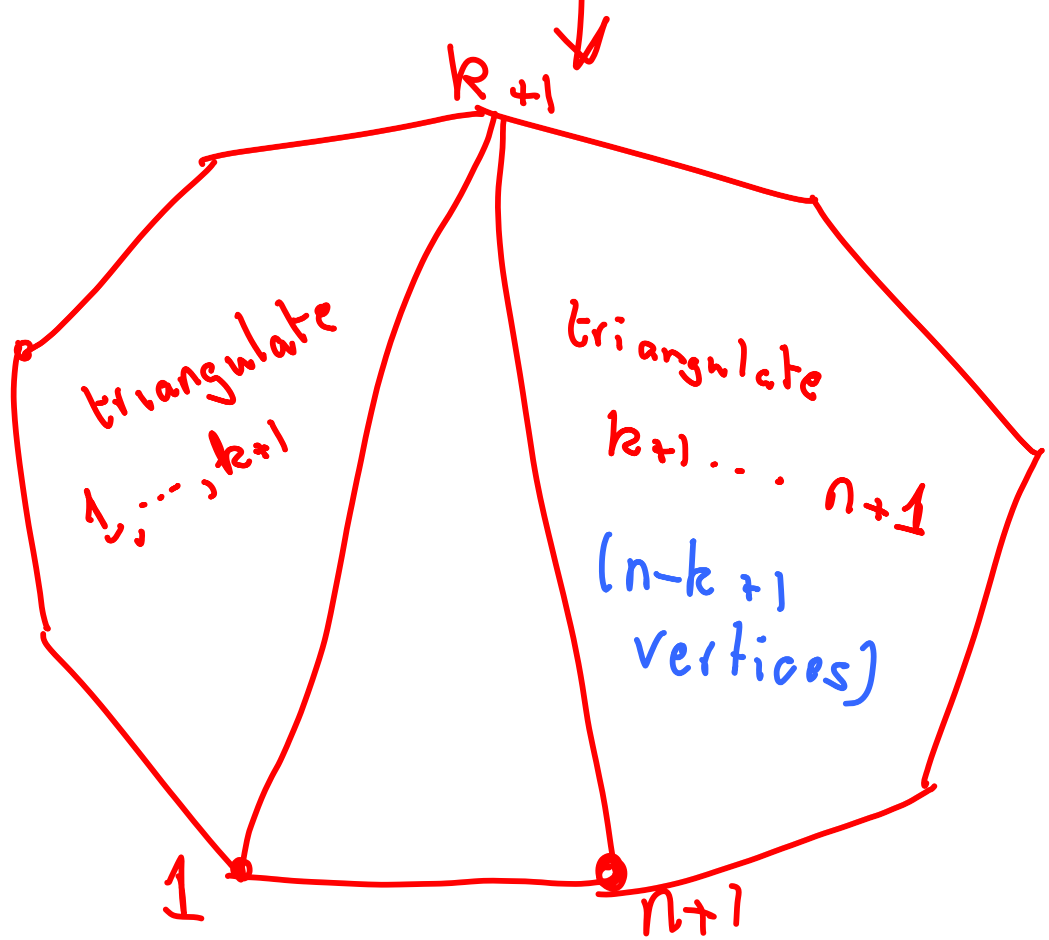
a_n = # of triangulations of P_{n+1}

$$a_n = \sum_{k=0}^{n} \underbrace{a_k \, a_{n-k}}$$

$n \geq 2$

$a_0 = 0$

$a_1 = a_2 = 1$



Multiply by x^n, $n \geq 2$ and

Sum: $a(x) = a_0 + a_1 x + a_2 x^2 + \dots$

$$\underbrace{\sum_{n=2}^{\infty} a_n x^n}_{a(x) - x} = \underbrace{\sum_{n=\cancel{2}\,0}^{\infty} \left(\sum_{k=0}^{n} a_k a_{n-k} \right) x^n}_{a(x)^2}$$

$$a(x) - x = a(x)^2$$

Add 0, 1 term

: $a_0 a_0$ 0 term

$a_0 a_1 + a_1 a_0$ 1

$$a(x)^2 - a(x) + x = 0$$

$$a(x) = \cancel{\frac{1+\sqrt{1-4x}}{2}} \quad \text{or} \quad \frac{1-\sqrt{1-4x}}{2}$$

We know that $a(0) = a_0 = 0$

So

$$a(x) = \frac{1}{2} - \frac{1}{2}\sqrt{1-4x}$$

$$a(x) = \frac{1}{2} - \frac{1}{2}\sqrt{1-4x}$$

$$= \frac{1}{2} - \frac{1}{2} \cdot \sum_{n=0}^{\infty} \binom{1/2}{n} (-1)^n (4x)^n$$

$$= \cancel{\frac{1}{2}} - \frac{1}{2}\left(\cancel{1} + \sum_{n=1}^{\infty} (-4x)^n \frac{\frac{1}{2}\left(\frac{1}{2}-1\right)\cdots\left(\frac{1}{2}-n+1\right)}{n!}\right)$$

$$= -\frac{1}{2}\sum_{n=1}^{\infty} \left((-x)^n \frac{2 \times (-2) \times (-6) \times \cdots \times (2-4n+4)}{n!}\right)$$

$$= \frac{1}{2}\sum_{n=1}^{\infty} x^n \frac{2 \times 2 \times 6 \times 10 \times \cdots \times 4n-6}{n!}$$

$$= \frac{1}{2}\sum_{n=1}^{\infty} x^n \frac{2 \cdot 2 \times 6 \times 10 \times \cdots \times 4n-6}{n!}$$

$$= \sum_{n=1}^{\infty} x^n \frac{1 \times 2 \times 6 \times 10 \times \cdots \times 4n-6}{n!}$$

$$= \sum_{n=1}^{\infty} x^n \frac{1 \times 2^{\times 4} \times 6^{\times 8} \times 10^{\times 12} \times \cdots \times 4n-6^{\times 4n-4}}{n!} \quad 4 \times 8 \times 12 \times \cdots 4n-4$$

$$= \sum_{n=1}^{\infty} x^n \frac{\cancel{2^{2n-2}}\,(2n-2)!}{n!\,\cancel{4^{n-1}}\,(n-1)!}$$

Discrete Probability

Ω in finite or countable

Ω Omega

$P : \Omega \to \mathbb{R}^+$

$$\sum_{w \in \Omega} P(w) = 1$$

if $w \in \Omega$

$P(w)$ is called "the probability" of w.

$$\Omega = \{H, T\}$$ Coin Toss

$$P(H) = \frac{1}{2}$$

$$P(T) = \frac{1}{2}$$

Rolling Dice

$$\Omega = \{1, 2, \ldots, 6\}$$

$$P(i) = \frac{1}{6}, \quad i = 1, 2, \ldots, 6$$

Both examples: Uniform Distribution:

$$P(\omega) = \frac{1}{|\Omega|}$$

$|\Omega| < \infty$

$\Omega = \{1, 2, \ldots \}$

Toss a coin until we get a head

E.G.

$\hat{\Omega} = \{H, TH, TTH, TTTH, \ldots \}$

$p = \frac{1}{2}$ $\quad p = \frac{1}{4}$ $\quad \frac{1}{8}$

$\Omega = \{1, 2, 3, \ldots \}$

Geometric Distribution:

1	2	3	4	...
p	$(1-p)p$	$(1-p)^2 p$	$(1-p)^3 p$	

p = probability of success.