

8/23/10

Count mappings:

$[m] = \{1, 2, \dots, m\}$

How many functions are there from

$[n] \rightarrow [m]$

Answer:  $m^n$

Choose  $f$ : #choices for  $f(1)$  is  $m$

#choices for  $f(2)$  is  $m$

#choices for the pair  $f(1), f(2)$  is  $m^2$

$\vdots$   
#choices for  $f(1), f(2), \dots, f(n)$  is  $m^n$ .

This also counts the  
number of sequences

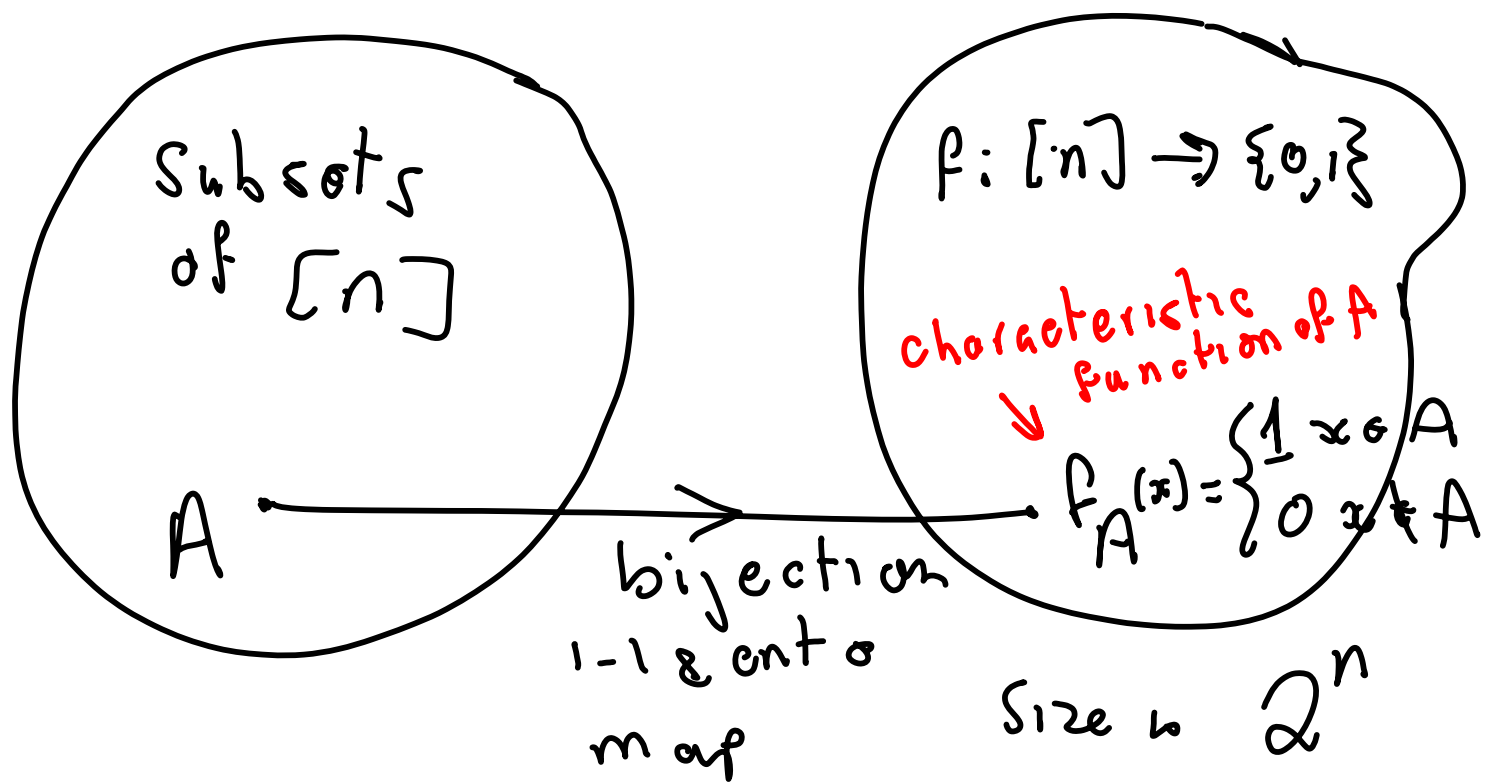
$$x_1, x_2, \dots, x_n$$

where  $x_i \in [m]$

$$x_i \equiv f(i)$$

$\psi(n) = \# \text{ of subsets of } [n]$

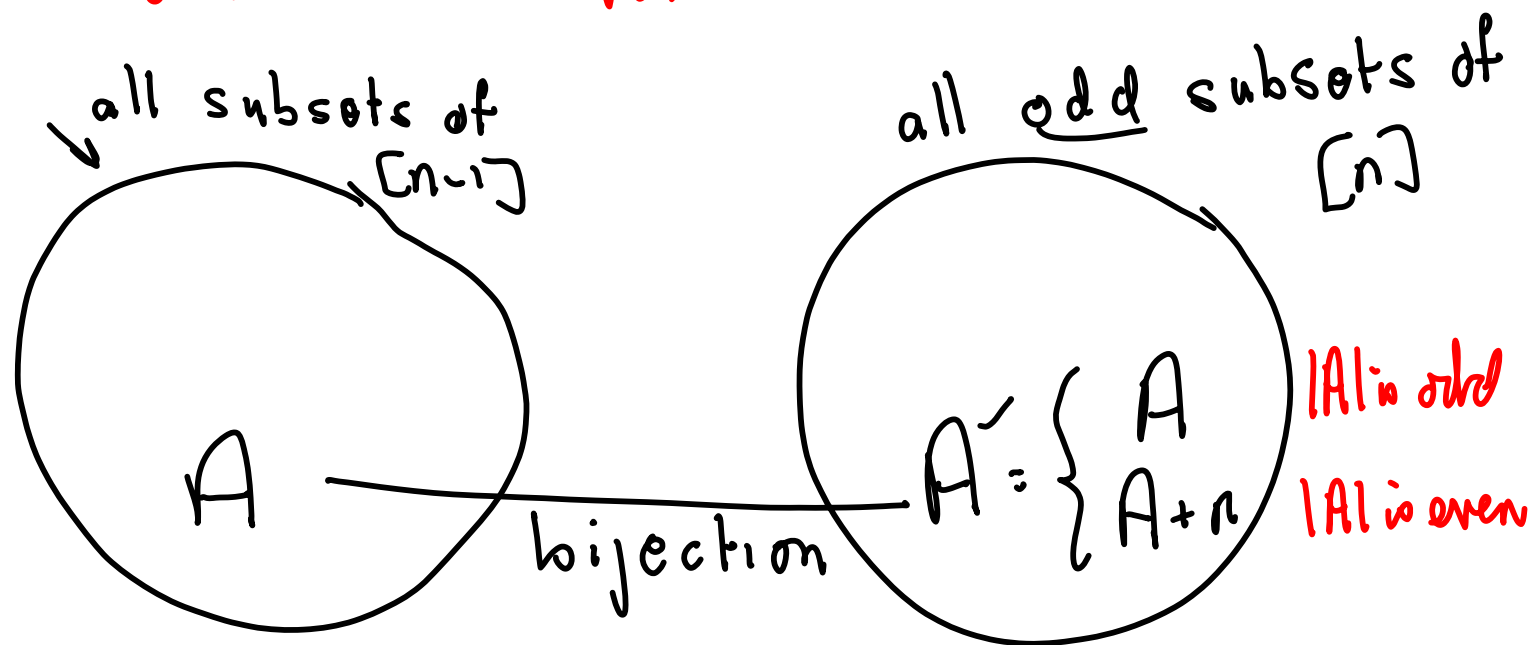
$$\psi(n) = 2^n$$



$\Psi_{\text{odd}}(n) = \# \text{ subsets of } [n]$   
that are odd

$\Psi_{\text{even}} = \# \dots \text{ even}$

$$\Psi_{\text{odd}}(n) = \Psi_{\text{even}}(n) = 2^{n-1}$$



How many 1-1 mappings from

$$[n] \longrightarrow [m]$$

$m$  choices for  $f(1)$

given  $f(1)$ , there are  $m-1$  choices for  $f(2)$

$\vdots$

given  $f(1), \dots, f(k)$ , there are  $m-k$   
choices for  $f(k+1)$

$$\# \text{ 1-1 functions} = m(m-1) \dots (m-n+1)$$

This also counts

# of sequences

$x_1, x_2, \dots, x_n$

where  $x_i \in [m]$

and  $x_i \neq x_j \quad \forall i, j$

$m = n$  :  $\# = n! = \underline{\# \text{ permutations}}$