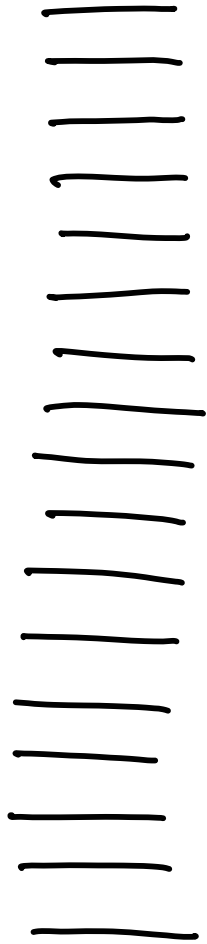


11\3\11



17 chips
Player A
should
win

Player who removes
last chip is the winner.

Two players

A & B

Player A goes
first.

In a move a player
takes some chips.
Suppose number of chips
a player can take is
1, 2, 3 or 4

N position : winning position
Next player should
win.

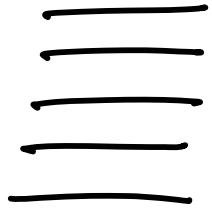
P: position : losing position
Previous player should
win.

Part of the problem is to determine
which positions are N & which are P.

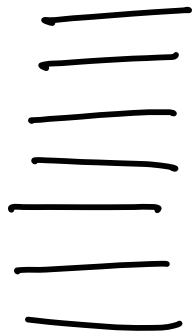


N position

Take all 3



P position



N position

Take 2

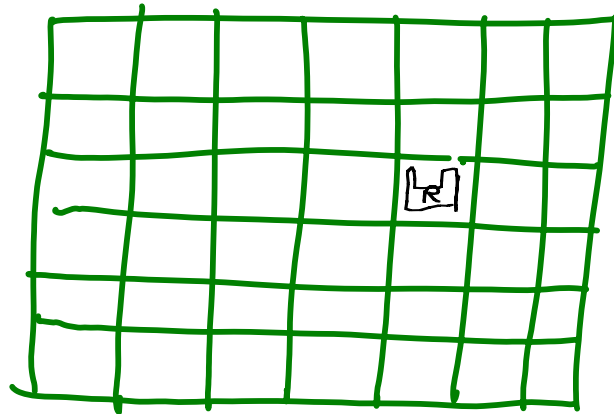
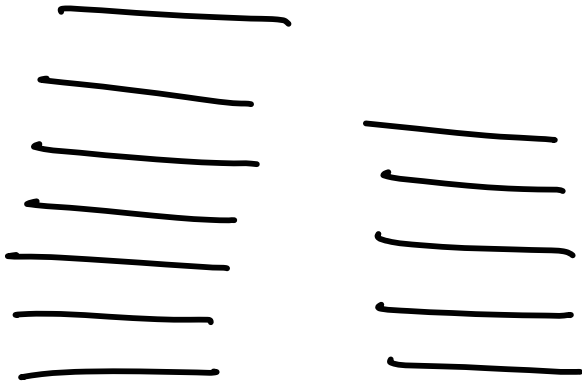
In general

n is a N -position if

$$n \bmod 5 \neq 0$$

Nim

2-pile game

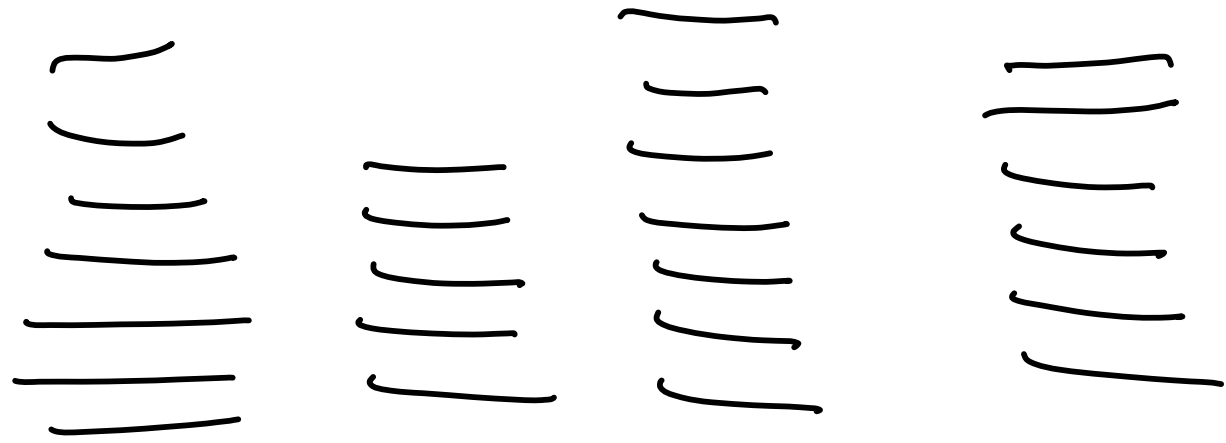


Player can take
any number, from
any non-empty
pile

A player can
move down or to
the left.

P on
diagonal



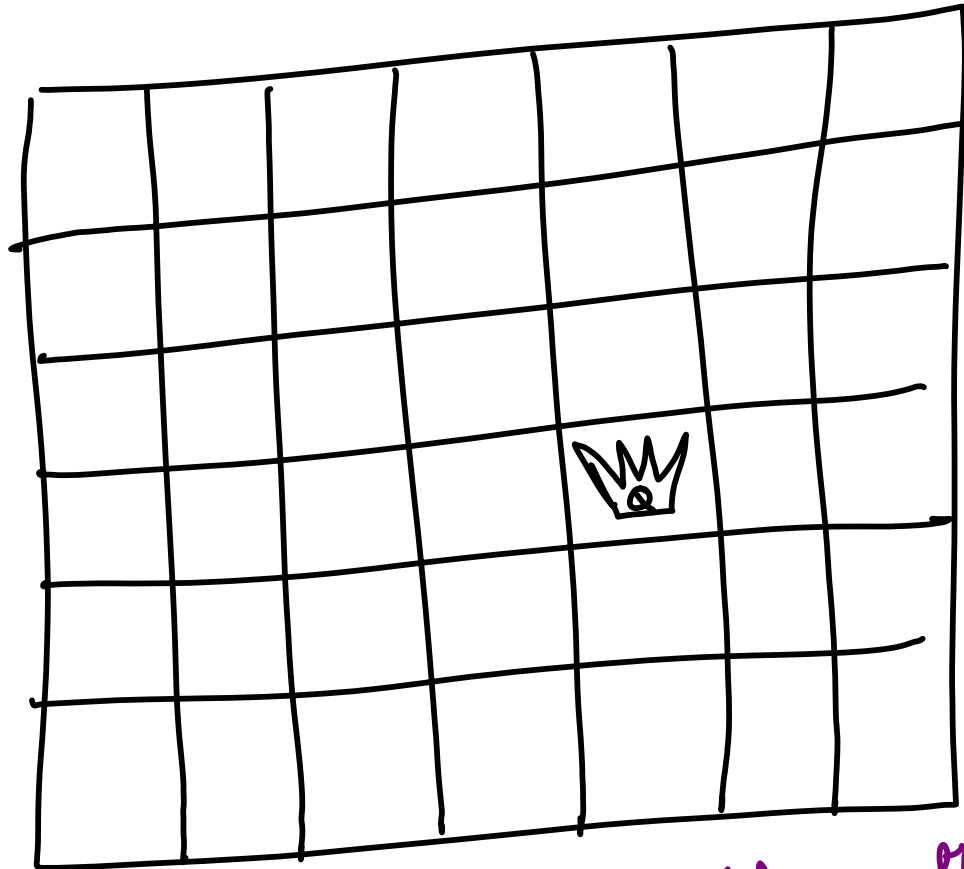


More complicated

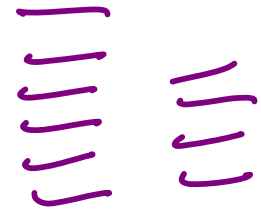
We will do this later

NIM

Wythoff's Nim

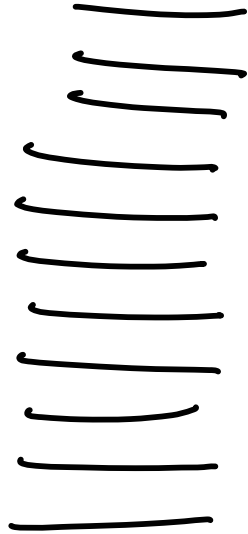


Move,
down and
to the left.



Take anything from 1 pile
or same from both piles

Another take away game



Rule: if last person
looks at, you
can only take

$$y \leq x.$$

Geography (2-player)

A: USA = U... of -- - a

B: Albania:

A: Switzerland

A: Austria:

B: Denmark

B: Argentina-

A: Kazakhstan

A: Australia

B: Nigeria

B: Azerbaijan:

A: Afghanistan

A: Netherlands

B: Norway

A: Yemen

B: Nicaragua

A: Angola

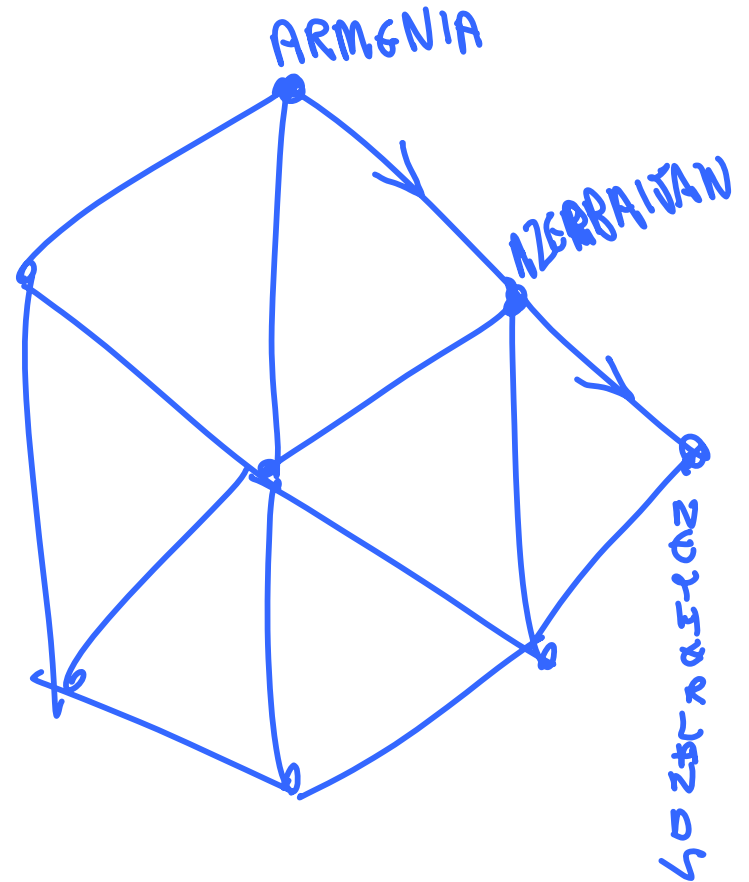
B: Armenia

A: Algeria

B: Andorra

A: Antigua

⋮



More: traverse on edge; delto a vertex

1	2	3
4	5	6
7	8	9

Abstraction:

Lines: $\{1, 2, 3\}$, $\{4, 5, 6\}$, $\{7, 8, 9\}$
 $\{1, 4, 7\}$. . . $\{1, 5, 9\}$

2 players X colors numbers Red
 O colors numbers Blue

A player wins if they can mono-color a set.
 Generalisation: Take any collection of sets.

Abstraction of all these
games:

Digraph $D = (V, A)$

