11\17\10

$$\nu_{X,G} = \frac{1}{|G|} \sum_{x \in X} |S_x|$$

For $g \in G$ let

$$\mathrm{Fix}(g) = \{x : g * x = x\}$$

Frobenius, Burnside

$$\nu_{X,G} = \frac{1}{|G|} \sum_{g \in G} |\mathrm{Fix}(g)|$$

To prove

$$\sum_{x \in X} |S_x| = \sum_{g \in G} |\mathrm{Fix}(g)|$$

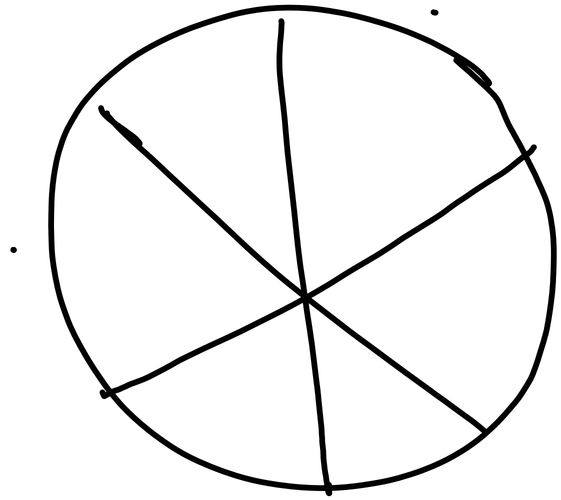
$$\begin{array}{cc} G \downarrow \backslash X \rightarrow & x \\ g & \left[ \begin{array}{c} \\ \mathbb{1}_{g * x = x} \\ \\ \end{array} \right] \end{array}$$

$\# 1\text{'s} = |\mathrm{Fix}(g)|$

$\# 1\text{'s} = |S_x|$

# Example 1

$e_i$ = rotate by $\frac{2\pi i}{6} = \frac{i\pi}{3}$



$e_0$: $|Fix(e_0)| = 64$

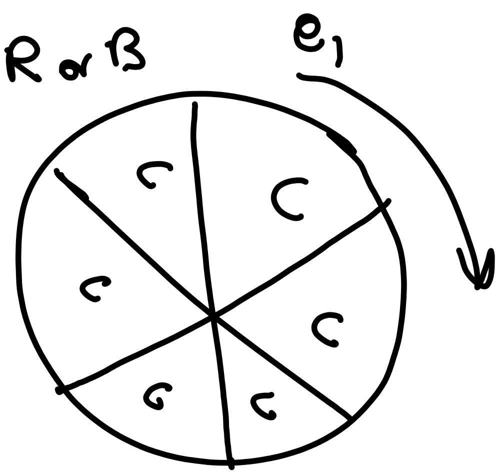
$e_1$: $|Fix(e_1)| = 2$

$e_2$: $|Fix(e_2)| = 4$

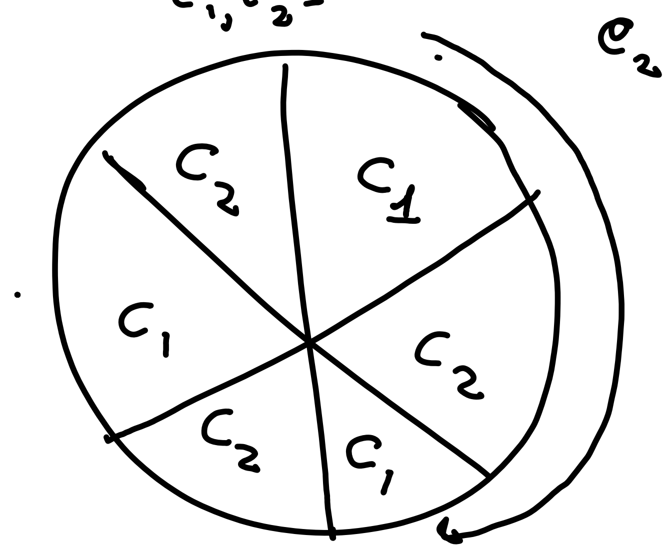
$\vdots$

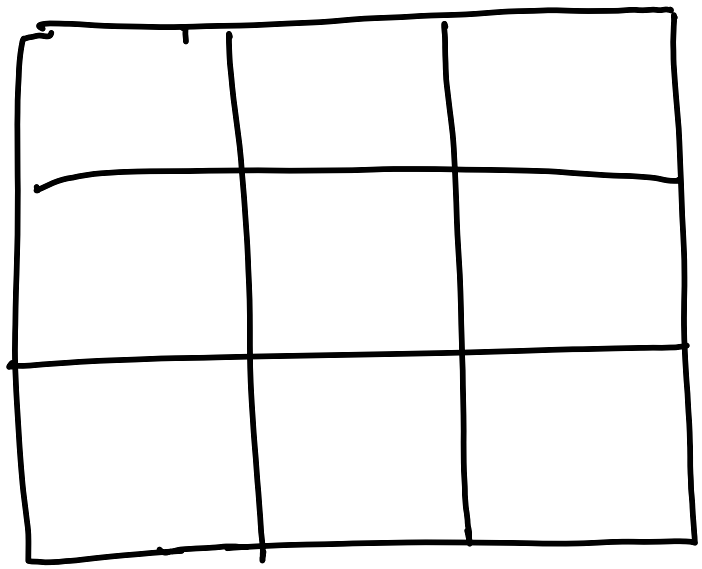
$|X| = 2^6 = 64$

$c = R$ or $B$




$c_1, c_2 = R$ or $B$





$|X| = 512$

$|Fux(\theta)| = 512$

$|Fux(a)| = 8$

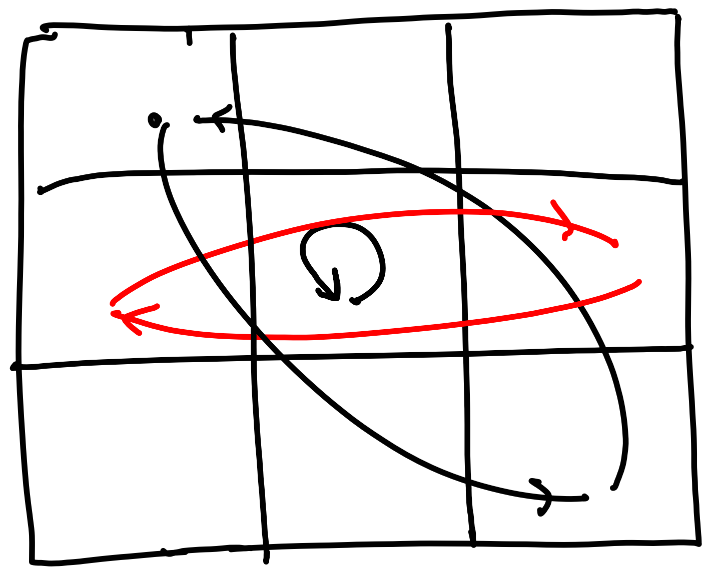
| | | |
|---|---|---|
| $c_1$ | $c_2$ | $c_1$ |
| $c_2$ | $c_3$ | $c_2$ |
| $c_1$ | $c_2$ | $c_1$ |

$|Fux(b)| = 32$

$\vdots$

| | | |
|---|---|---|
| $c_1$ | $c_2$ | $c_3$ |
| $c_4$ | $c_5$ | $c_4$ |
| $c_3$ | $c_2$ | $c_1$ |

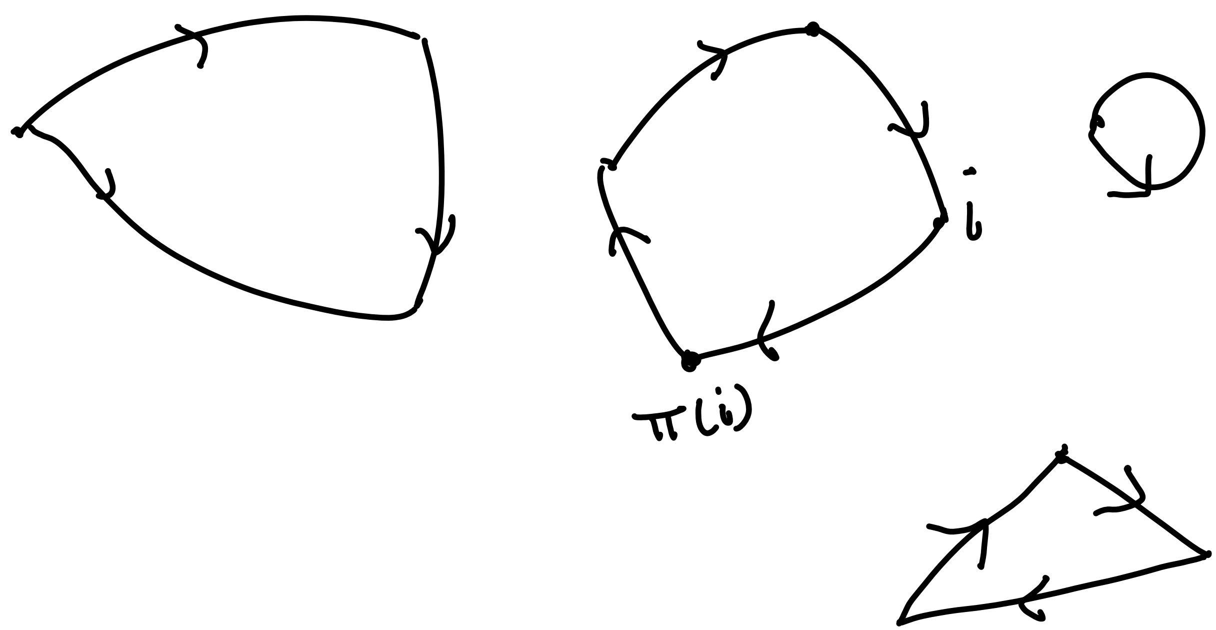
b/



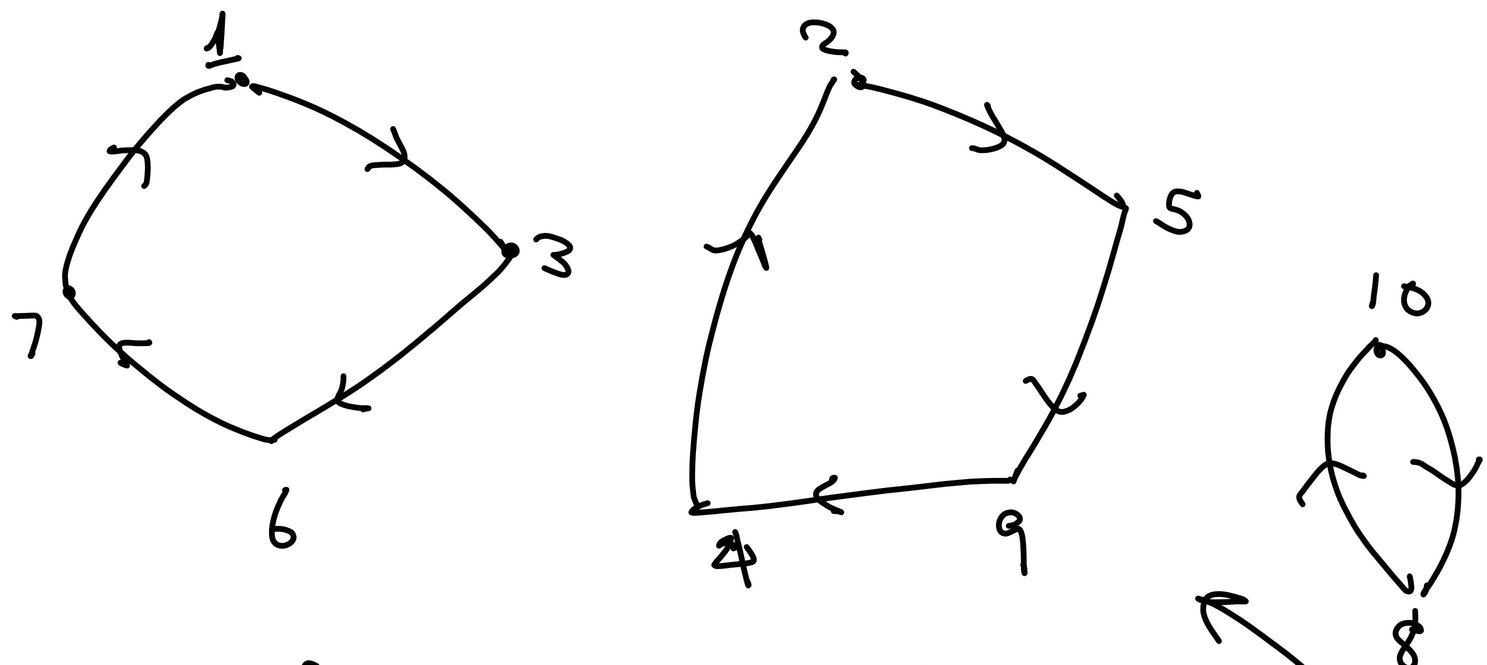
$X$ is the set of colorings of $D$

$\pi : D \longrightarrow D$ is a permutation

$\pi$ decomposes into cycles.



| $i$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| $\pi(i)$ | 3 | 5 | 6 | 2 | 9 | 7 | 1 | 10 | 4 | 8 |




If $\pi$ fixes a coloring then 1, 3, 6, 7 have the same color

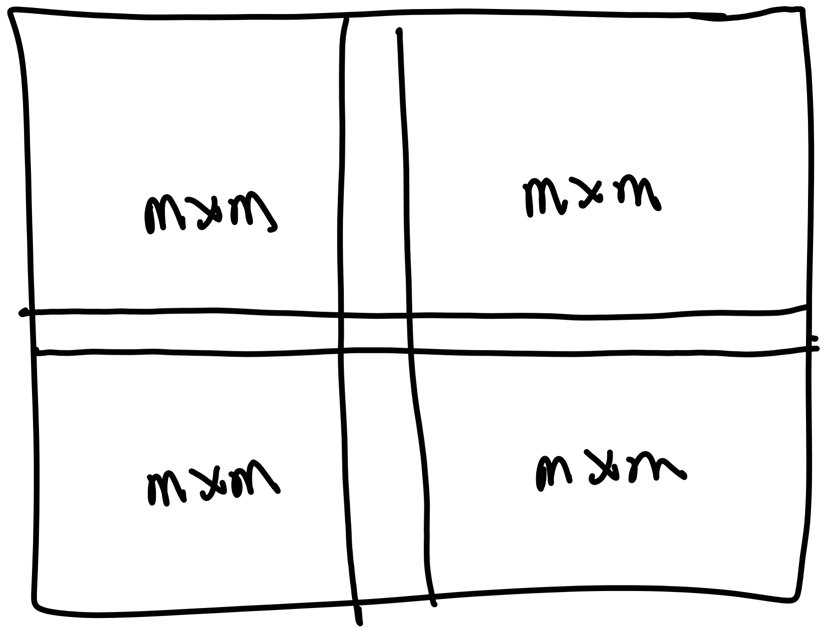
$|\mathrm{Fix}(\pi)| = 2^3$

To determine $|\mathrm{Fix}(g)|$

I have to compute # of cycles

in $g$.

$n \times n$ chessboard




$n = 2m+1$ is odd.

| | g | e | a | b | c | p | q | r | s |
|---|---|---|---|---|---|---|---|---|---|
| $\|Fix(g)\|$ | $2^{n^2}$ | $2^{m^2+m+1}$ | $2^{2m^2+2m+1}$ | | $2^{2m^2+3m+1}$ | | | | |

$1$ 1-cycle

$\frac{n^2-1}{4}$ 4-cycle

$n$ 1-cycles

$\frac{n^2-n}{2}$ 2-cycles