10/8/10

Suppose $|V| = n$, $|E| = n-1$ edges

(a) G is connected

$\Downarrow$

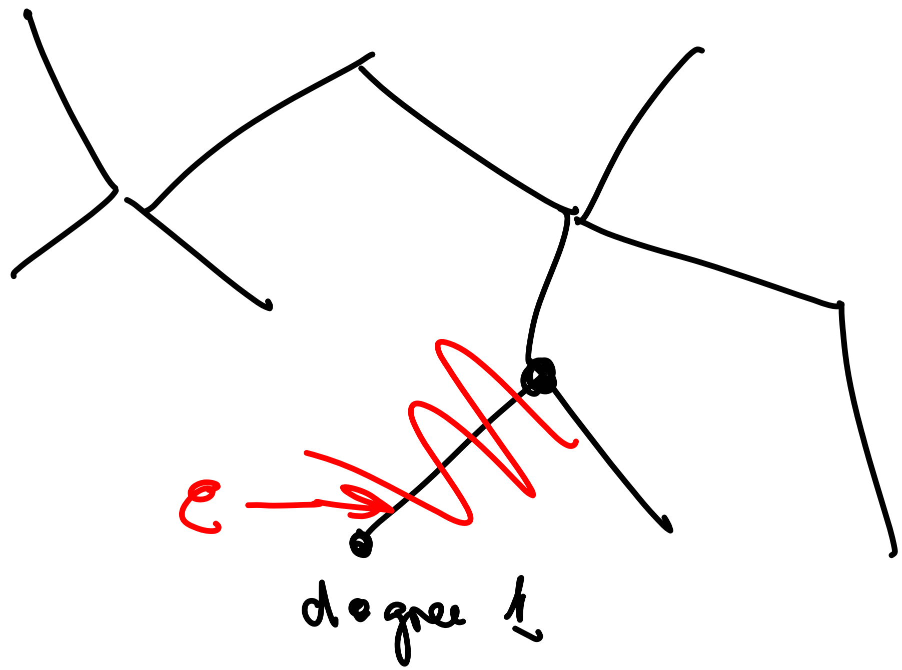
(b) G is acyclic

$\Downarrow$

(c) is a tree

build G one edge at at a time. We end with one component. We reduced # comps $n \longrightarrow 1$ in $n-1$ steps and so we could not have made a cycle

When we added edges, we did not make a cycle. So # comps went down by one each time. End with one comp



T has n vertices, $n-1$ edges

$T-e$ has $n-1$ vertices, $n-2$ edges

no cycle

$\Downarrow$

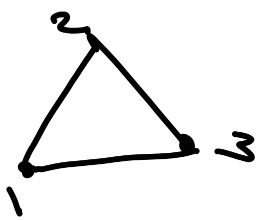
we still have a tree

Counting spanning trees of K_n

$n=2$

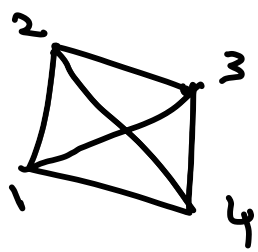
trees = 1

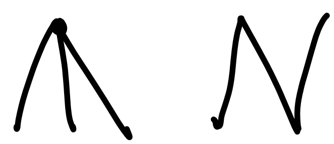
$n=3$



trees = 3

$n=4$

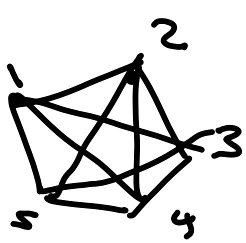


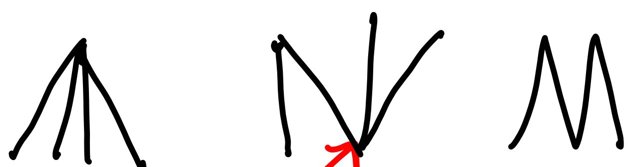


$4 \qquad \frac{4!}{2} = 12$

trees = 16

$n=5$





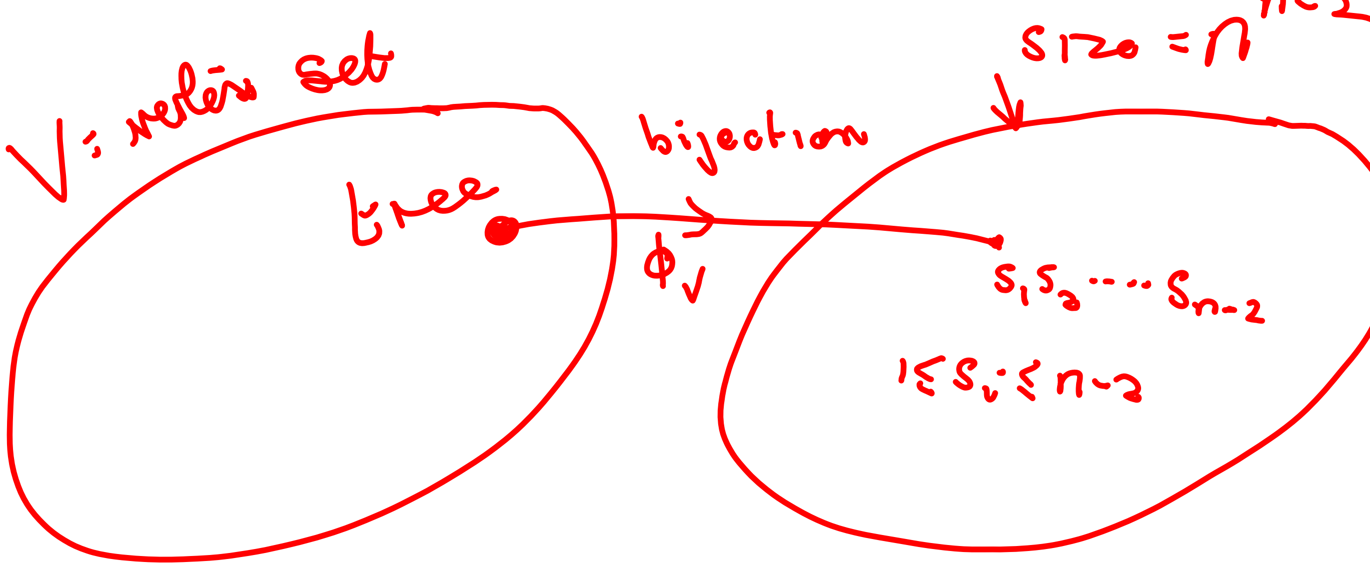
$5 \qquad 5 \times 4 \times 3 \qquad \frac{5!}{2}$

trees = 125

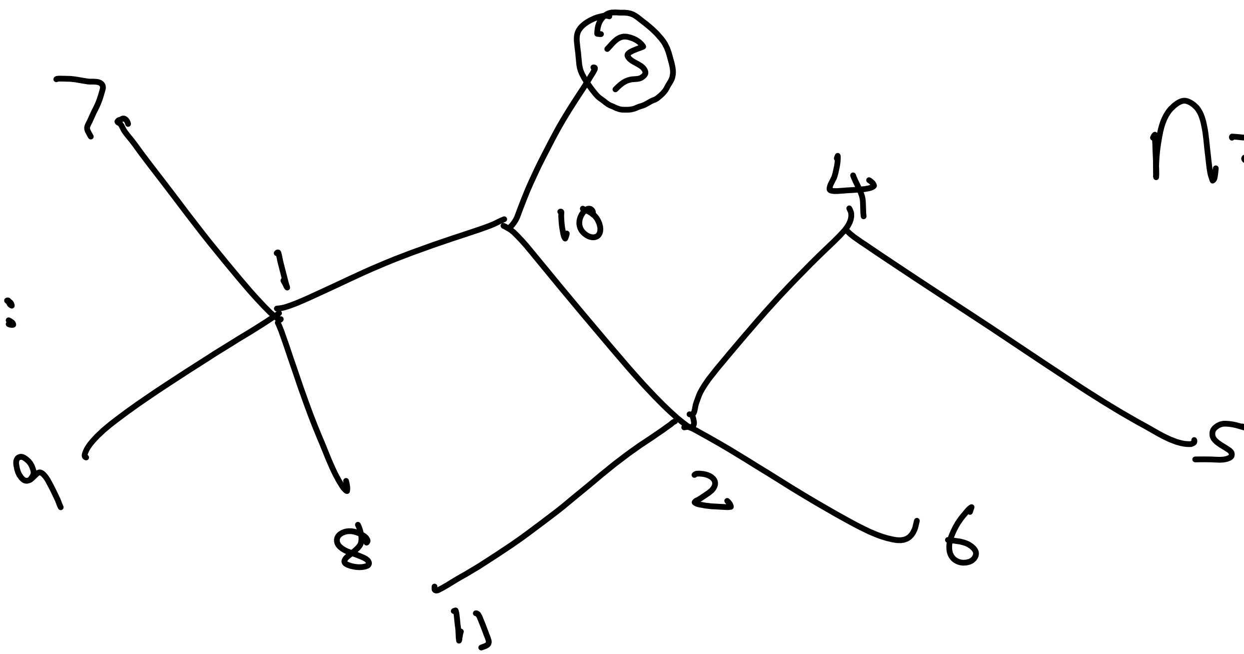
Cayley's formula:

$$\# \text{trees} = n^{n-2}$$

Prüfer Correspondence.

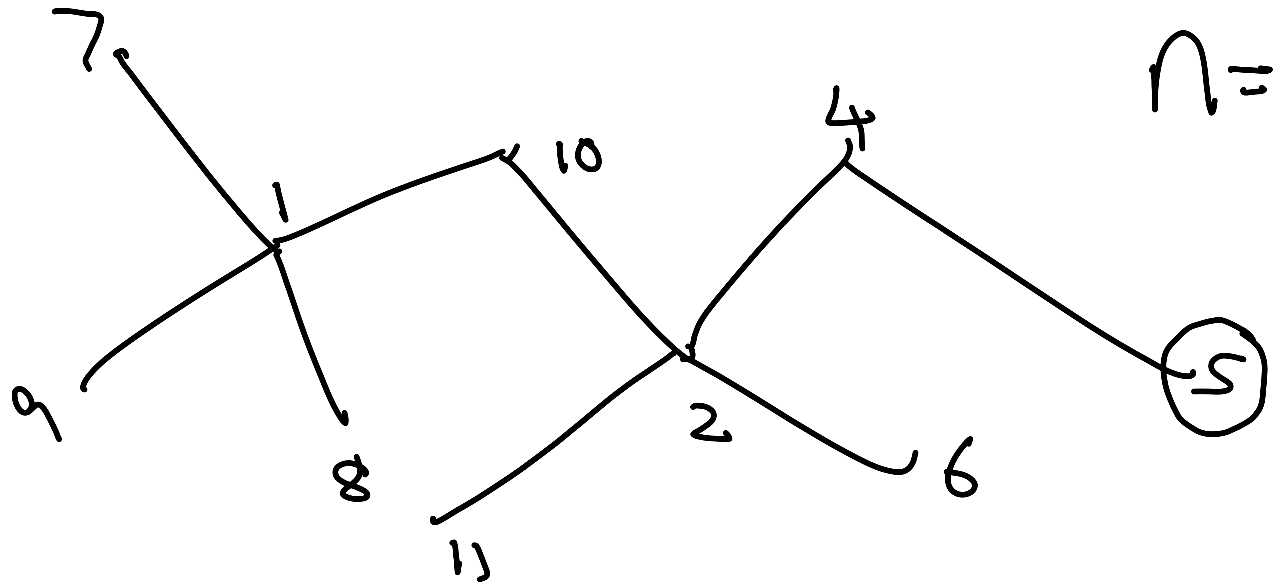


T:



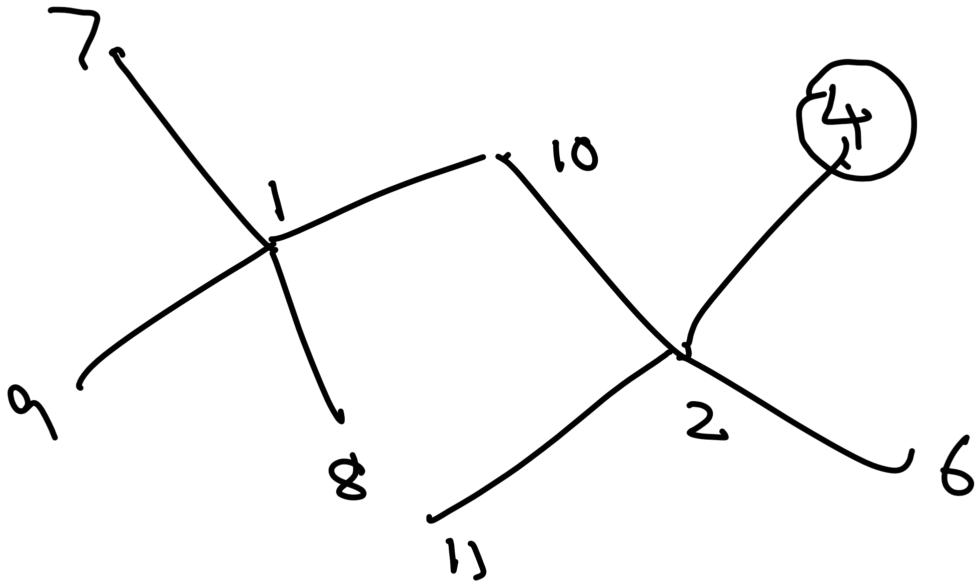
$n = 11$

$\phi(T) = 10,$



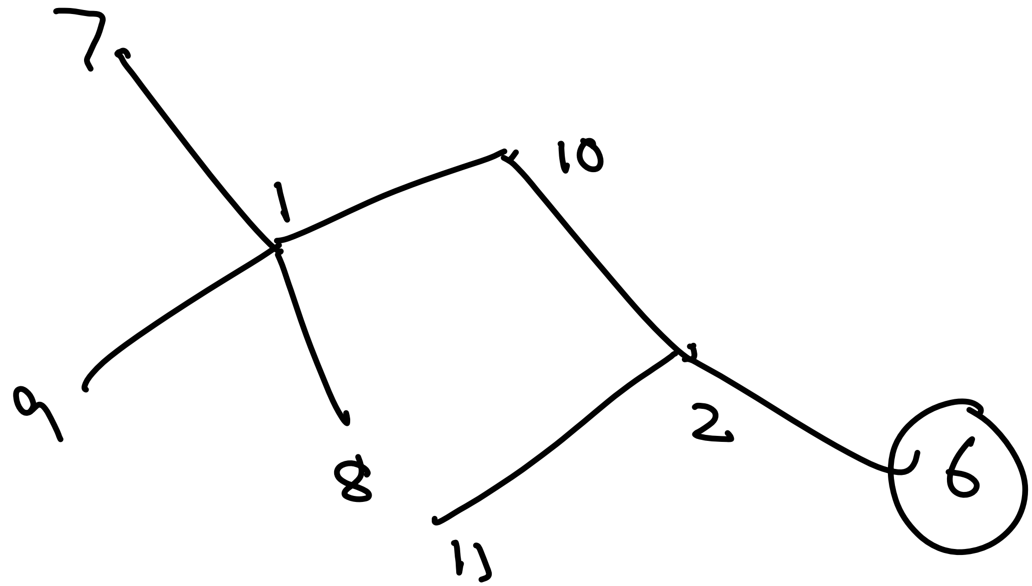
$n = 11$

$\phi(T) = 10, 4,$



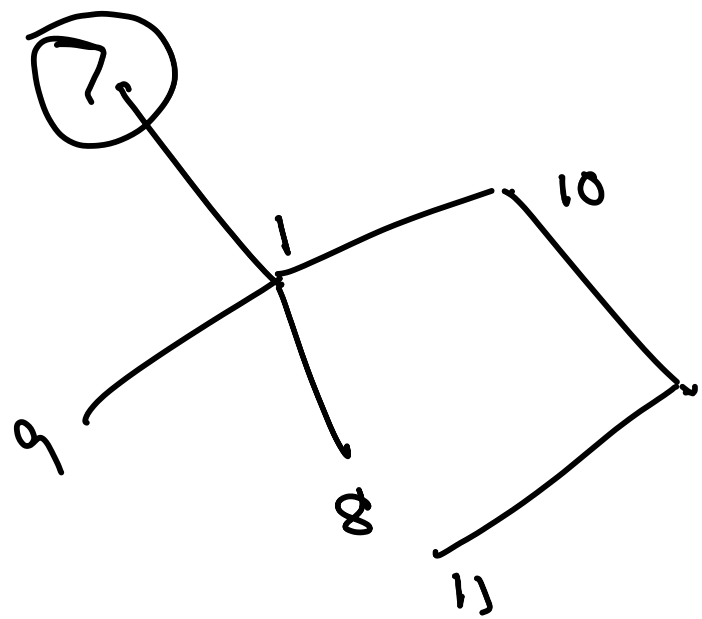
$n = 11$

$\phi(T) = 10, 4, 2$



$n = 11$

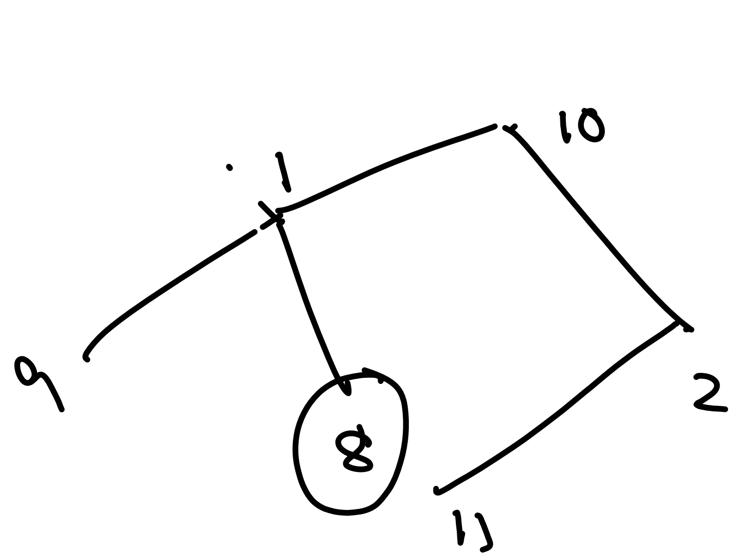
$\phi(T) = 10, 4, 2, 2,$



$n = 11$

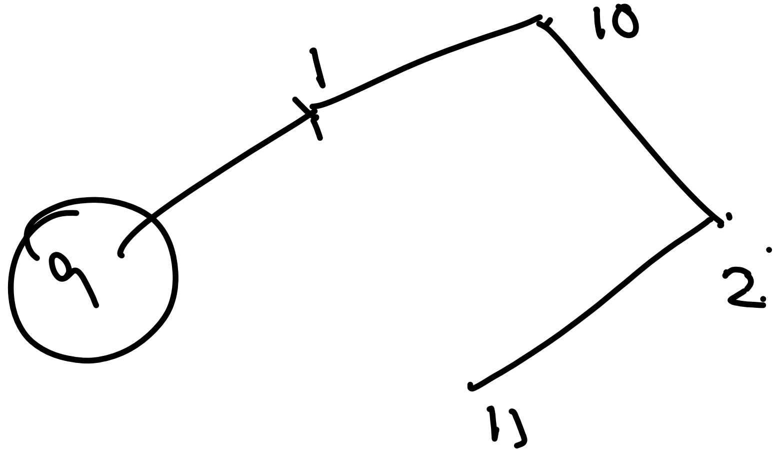
$\phi(T) = 10, 4, 2, 2, 1$

$n = 11$



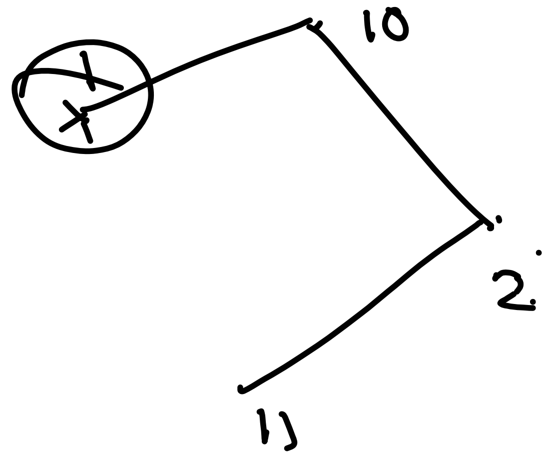
$\phi(T) = 10, 4, 2, 2, 1, 1,$

$n = 11$



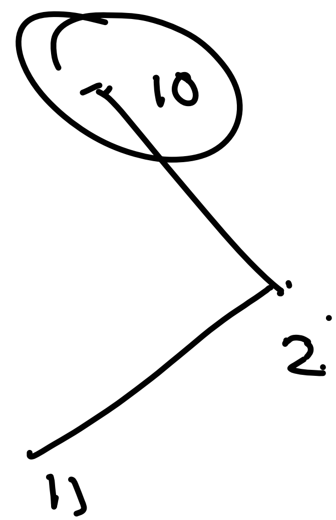
$\phi(T) = 10, 4, 2, 2, 1, 1, 1$

$n = 11$



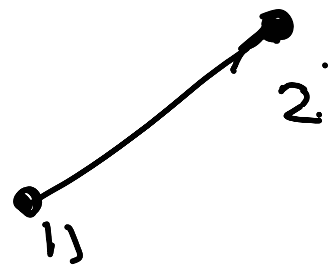
$$\phi(T) = 10, 4, 2, 2, 1, 1, 1, 10$$

$n = 11$



$\phi(T) = 10, 4, 2, 2, 1, 1, 1, 10, 2$

$n = 11$



$$\phi(T) = 10, 4, 2, 2, 1, 1, 1, 10, 2$$

So, given T, $\phi(T)$ is well defined.

Can one go back? i.e. given sequence, can I construct T?

YES

Does $\exists$ inverse function ϕ_V^{-1}?

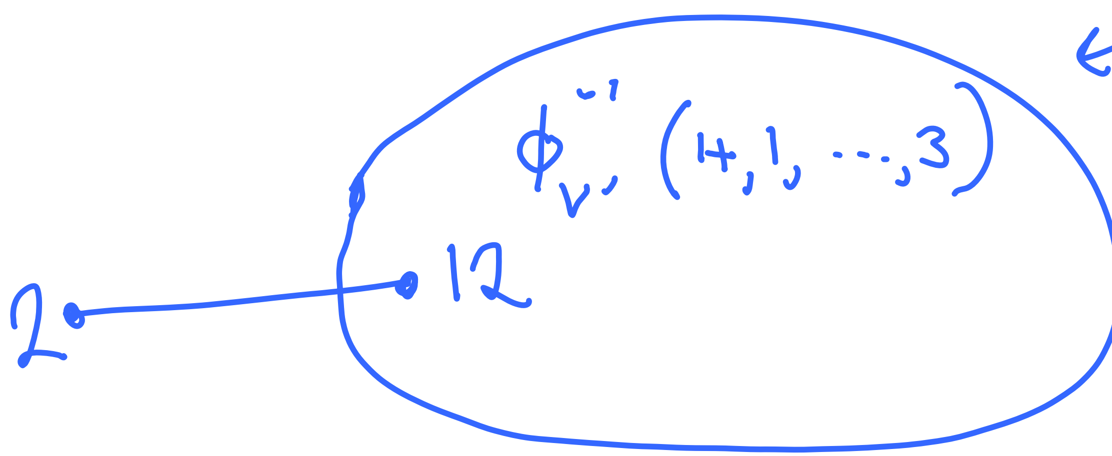
Proof is by induction on $|V|$.

Suppose $n = 12$ and sequence is

12, 4, 1, 3, 8, 8, 7, 6, 4, 3,

What was the first thing we did?

tree



$V = \{1, 2, \ldots, 12\}$

$V' = \{1, 3, 4, \ldots, 12\}$

Claim

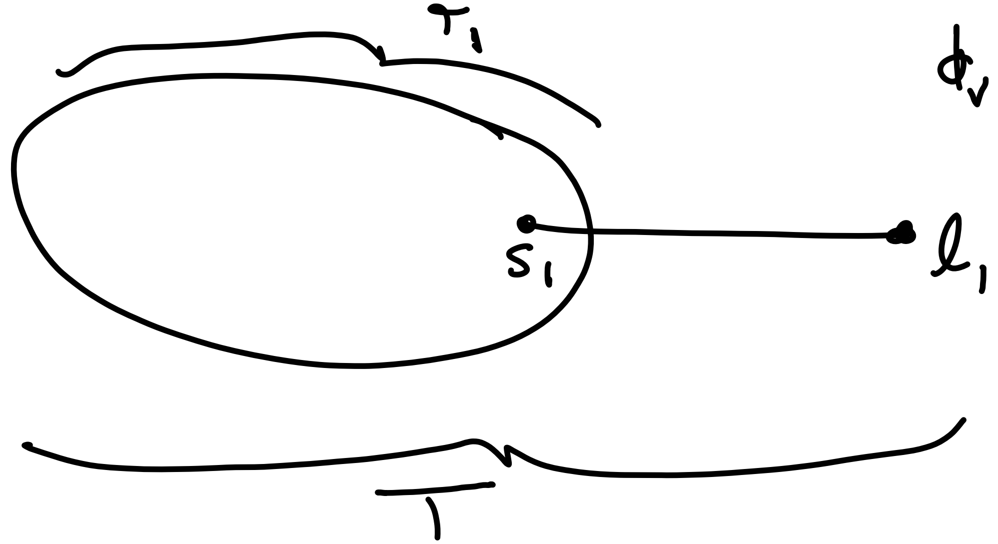
$v \in V(T)$ appears $d_T(v) - 1$ times.

By induction on $|V|$.

$n=2$ $\quad$ Λ empty string

Assume true for $n-1 \geq 2$

$\phi_v(T) = s_1 \phi_{v_1}(T_1)$



# of trees where vertex i

has degree d_i

$=$ # sequences where i

appears $d_i - 1$ times

$$= \binom{n-2}{d_1 - 1, d_2 - 1, \ldots, d_n - 1}$$