10\29\10

$a_1, a_2, a_3, \dots \dots a_{k^2+1}$

Show $\exists$ monotone sequence of length $k+1$.

For given i let

$a_i, a_i^1, a_i^2, \dots a_i^{l-1}$

a longest monotone increasing sequence starting at a_i

6, 5, 8, 3, 1, 10, 17

$i = 3$

a_i $\quad a_i^1$ $\quad a_i^2$

8 $\quad$ 10 $\quad$ 17

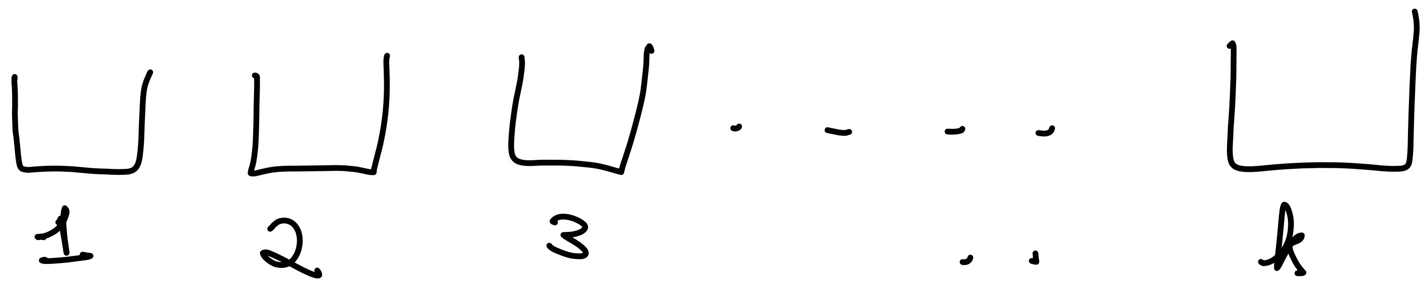
$l(i)$ = length of the sequence.

Two Cases:

(i) $\exists\, i : \ l(i) \geq k+1$ ✓

(ii) $l(i) \leq k, \quad \forall\, i$

put i into bucket $l(i)$



PHP $\Rightarrow \exists$ bucket with $\geq \left\lceil \dfrac{k^2+1}{k} \right\rceil = k+1$ elements.

$\geq k+1$ elements

$i_1 < i_2 < \cdots < i_{k+1}$

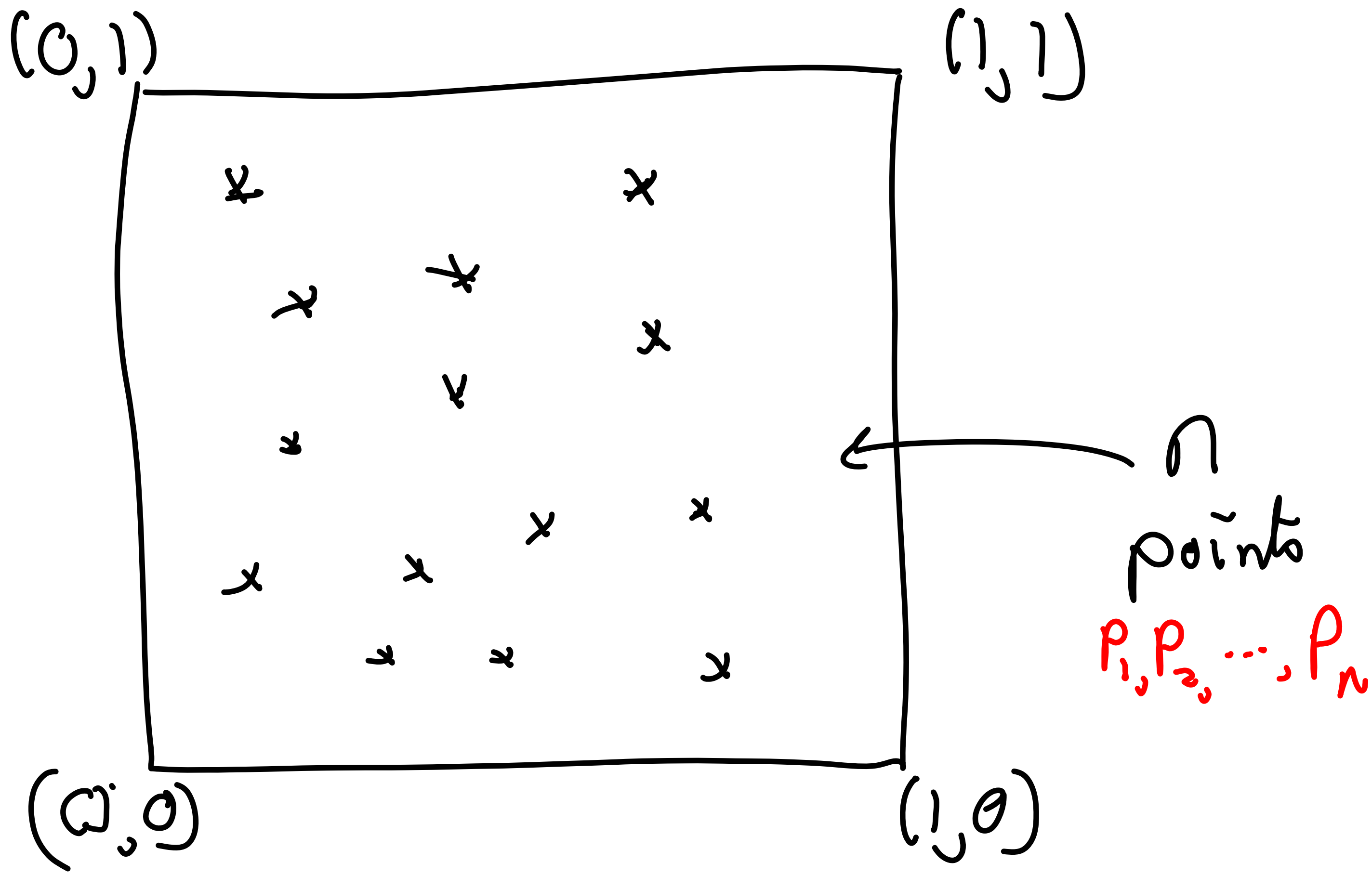
bucket b

Claim: $a_{i_1} \geq a_{i_2} \geq \cdots \geq a_{i_{k+1}}$

Suppose for example $a_{i_1} < a_{i_2}$

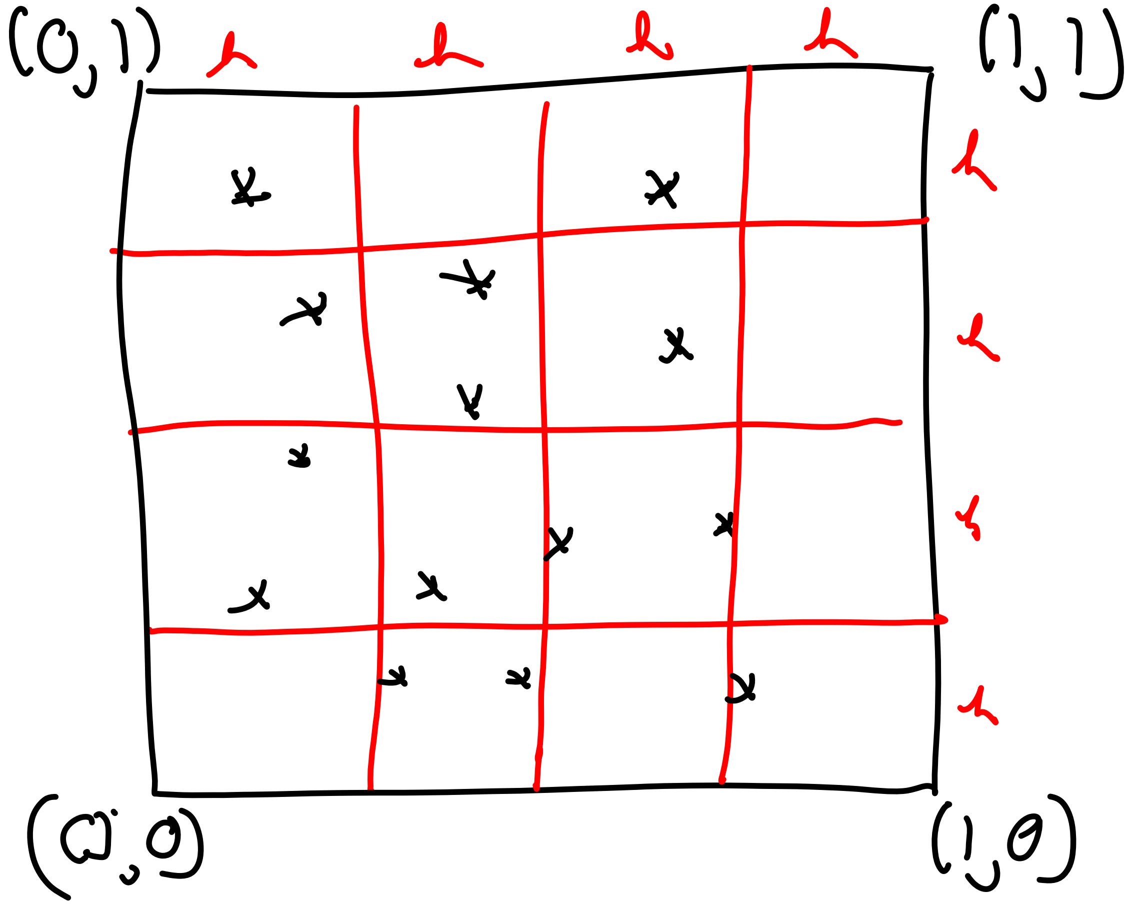
$\Rightarrow$ i_1 should be in a bucket with label $\geq b+1$

Simple Geometric Problem



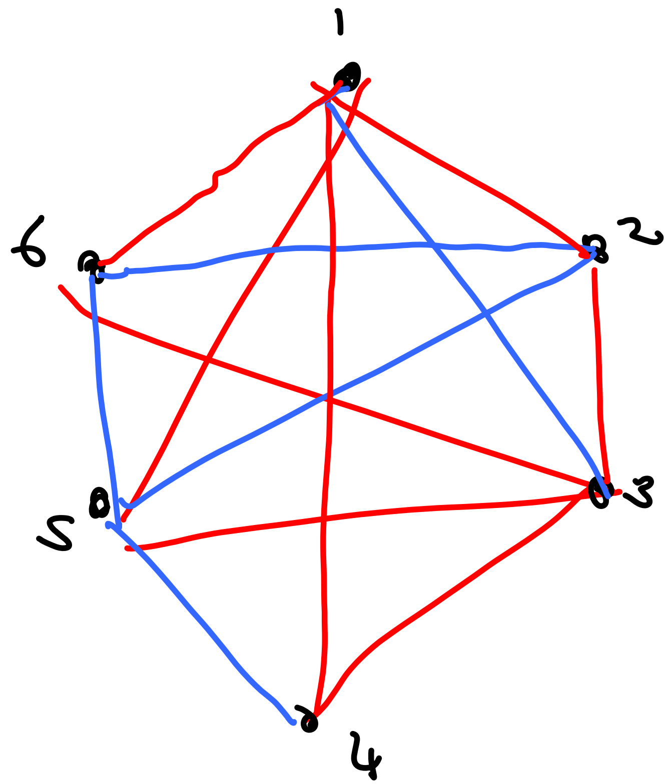
$\exists$ triangle $P_{i_1} P_{i_2} P_{i_3}$ $\ni$ area $\leq \sim \frac{1}{n}$

$\#\text{squares} = \frac{1}{h^2} < \frac{n}{2}$



Argue that some $\square$ contains ≥ 3 points. So area of $\Delta \leq \square \leq \frac{h^2}{2}$.

Ramsey Theory



6 people in a room

$\Downarrow$

either

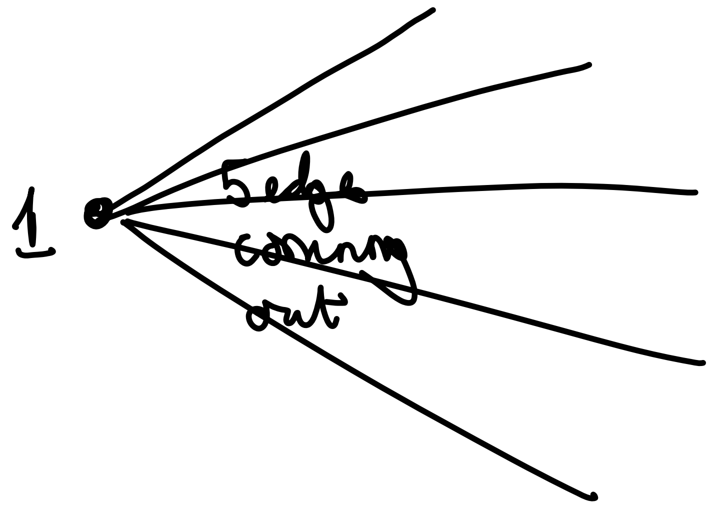
(i) $\exists$ 3 that like each other

or

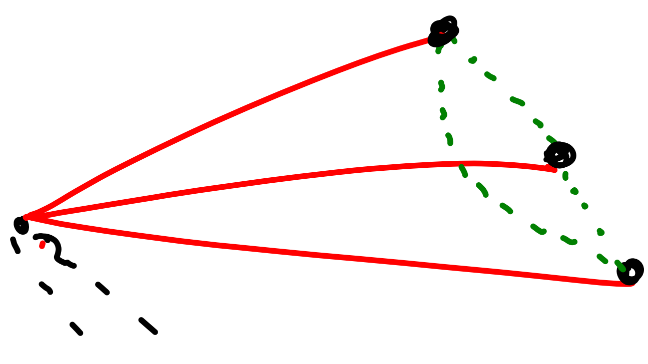
(ii) $\exists$ 3 that dislike each other

Every 2-coloring of the edges of K_6 yields a mono-chromatic Δ.

Proof for K_6



Some color is used ≥ 3 times, Red



Either all Blue – Blue Δ

or $\exists$ Red – Red Δ

Ramsey's Theorem

For any positive integers k, l

$\exists$ an integer $R = R(\overset{\text{Red}}{k}, \overset{\text{Blue}}{l})$

such that if we 2-color the edges of K_R then $\exists$ either a

Red K_k or a blue K_l.

[True for $N \geq R$]

$$R(3,3) = 6$$

$$R(\overset{R}{2}, \overset{B}{k}) = R(k,2) = k$$

If Red is used then we get Red K_2,
if Red is not used then we have a Blue K_k.

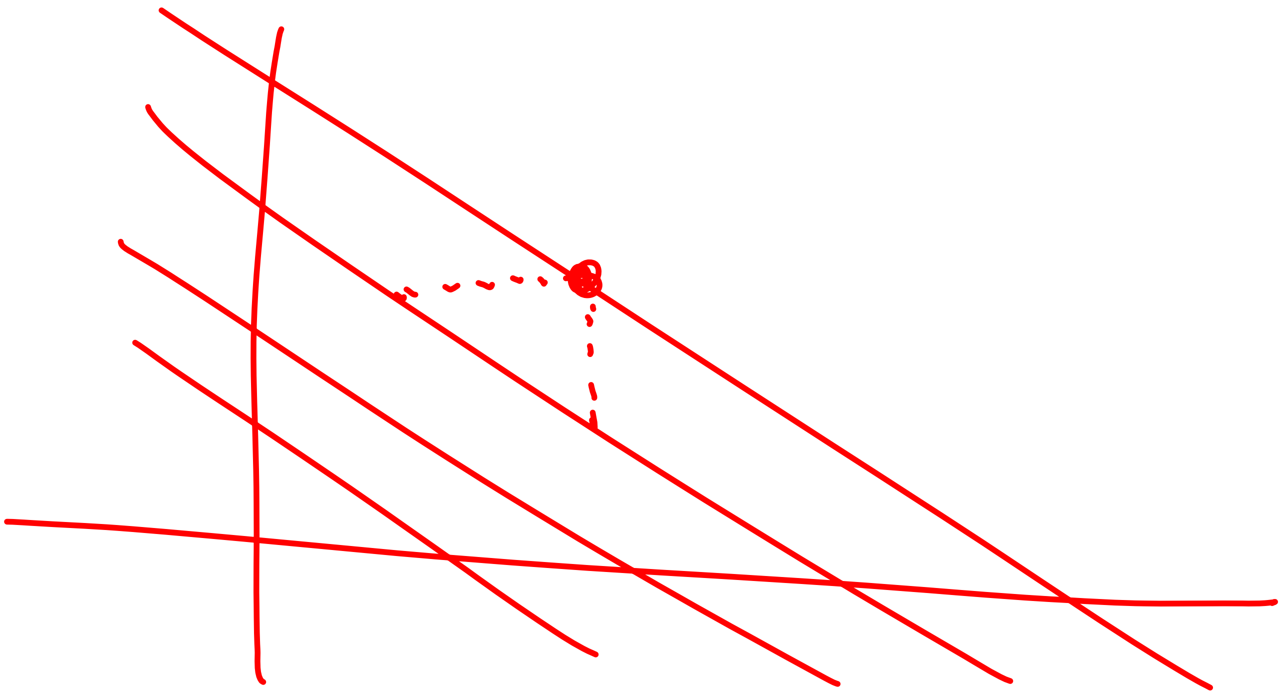
$$R(4,4) = 18$$

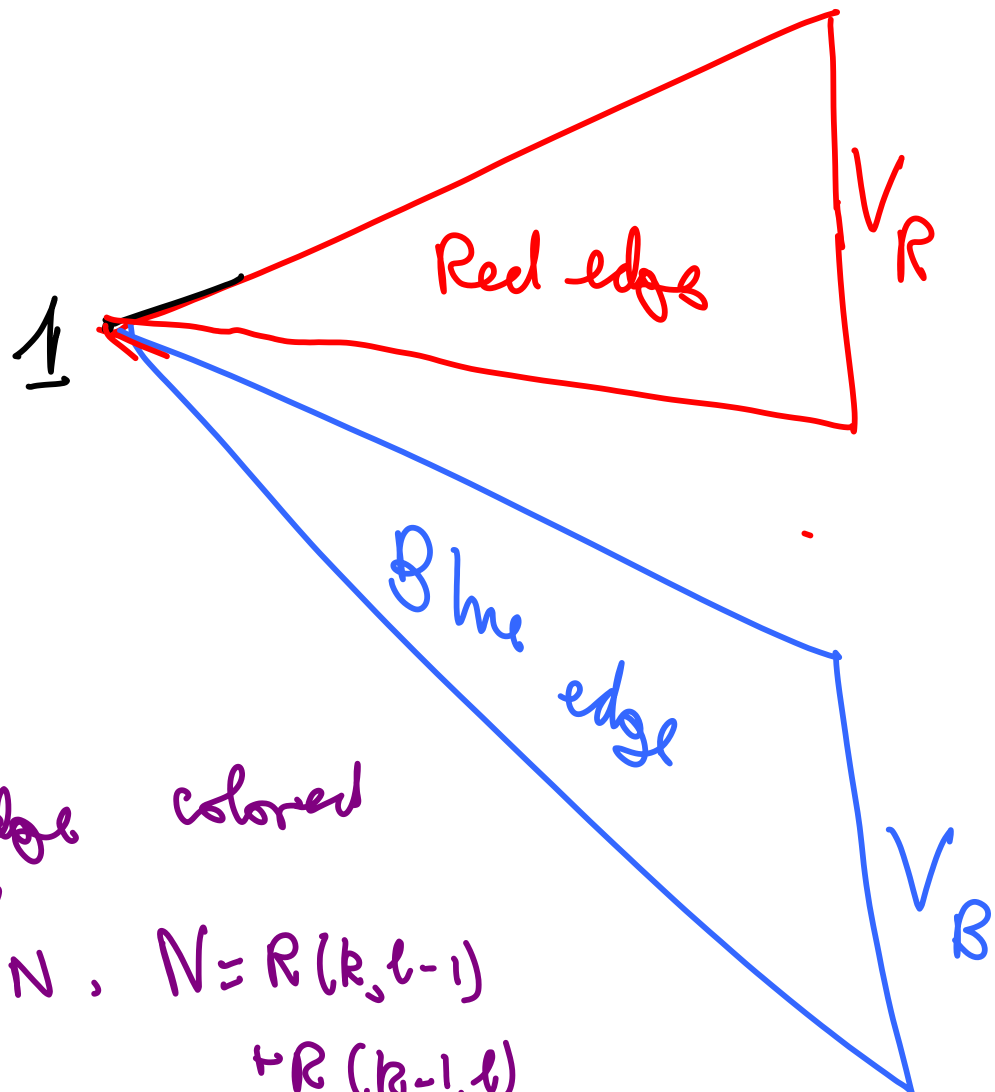
$$R(5,5) = ? \in [43, 49]$$

$$R(k,l) \leq R(k,l-1) + R(k-1,l)$$

If these 2 exists then

So does





Either

$|V_R| \geq R(k-1, \ell)$

or

$|V_B| \geq R(k, \ell-1)$

Edge colored

K_N, $N = R(k, \ell-1) + R(k-1, \ell)$