10\27\10

Erdös-Ko-Rado Thm

Theorem

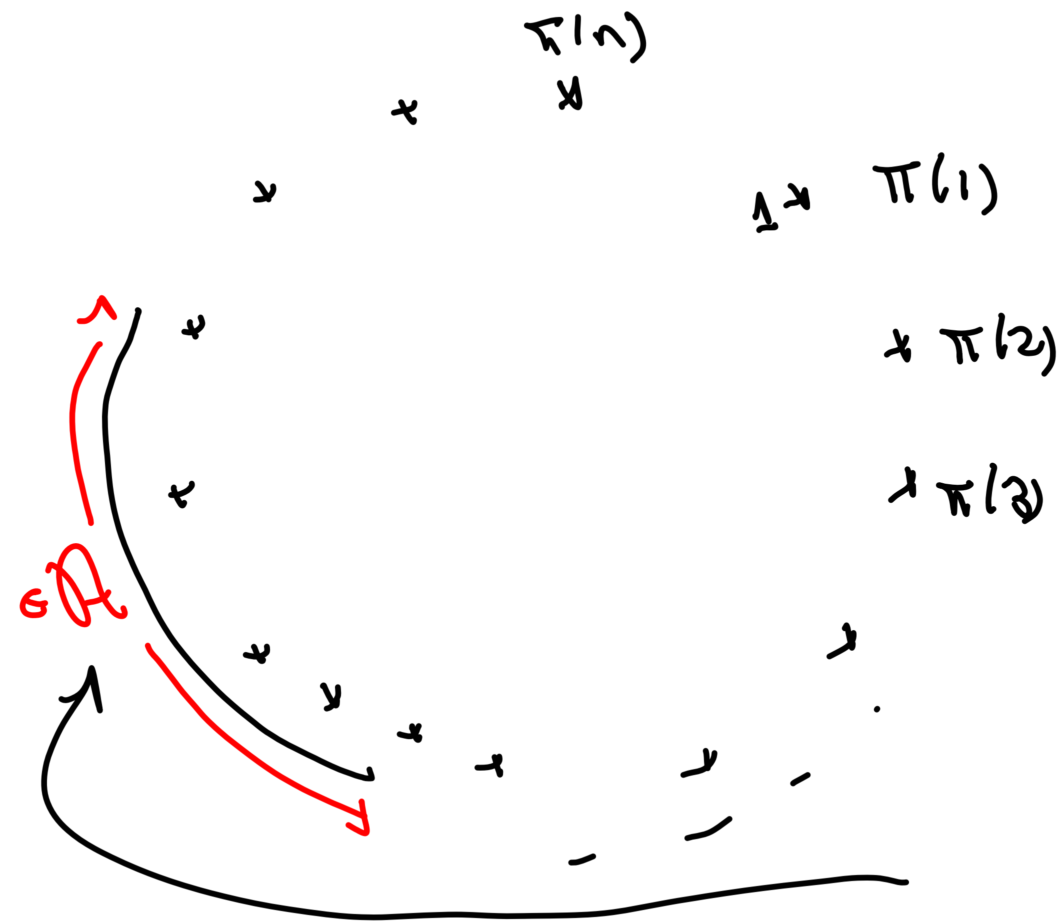
If $\mathcal{A}$ is an intersecting family and $\mathcal{A} \subseteq \binom{[n]}{k}$, $k \leq \lfloor n/2 \rfloor$

then

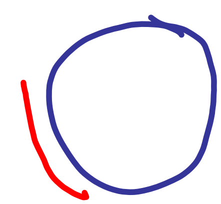
$$|\mathcal{A}| \leq \binom{n-1}{k-1}$$

$k > \lfloor n/2 \rfloor$ $\Rightarrow \binom{[n]}{k}$ is intersecting

Take all k-sets containing 1 then we get an intersecting family of size



Count number of incidence like this



$$\Theta(\pi, A) = \begin{cases} 1 & \text{somewhere} \\ 0 & \text{otherwise} \end{cases}$$

Claim: $\sum_{A \in \mathcal{A}} \theta(\pi, A) \leq k, \quad \forall \pi.$

Now let π be a random permutation.

$$\sum_{A \in \mathcal{A}} E(\theta(\pi, A)) \leq k$$

choices for θ

ordering of A

ordering of the rest

$$\frac{n \times k! \times (n-k)!}{n!}$$

permutations

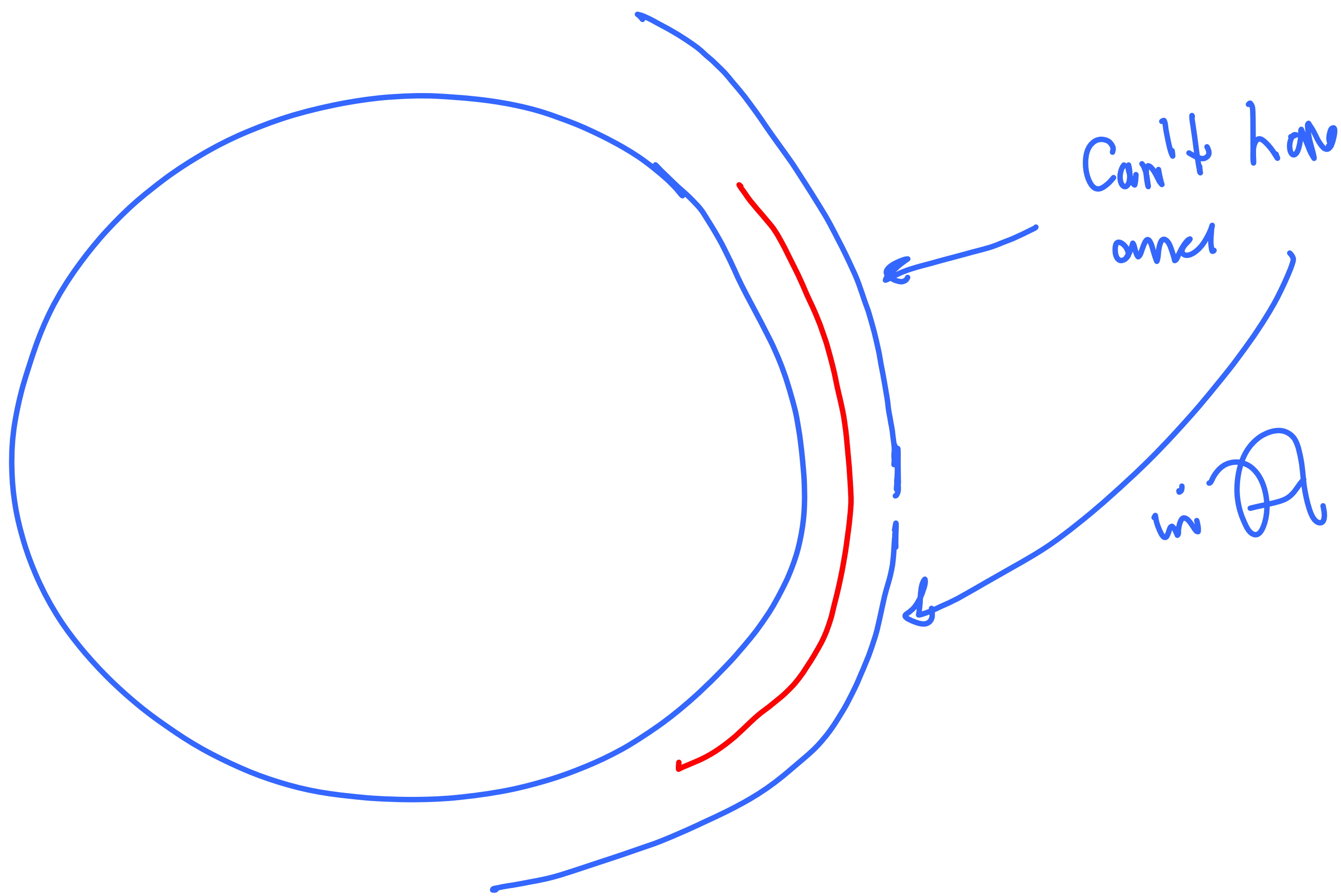
$\bigcirc$

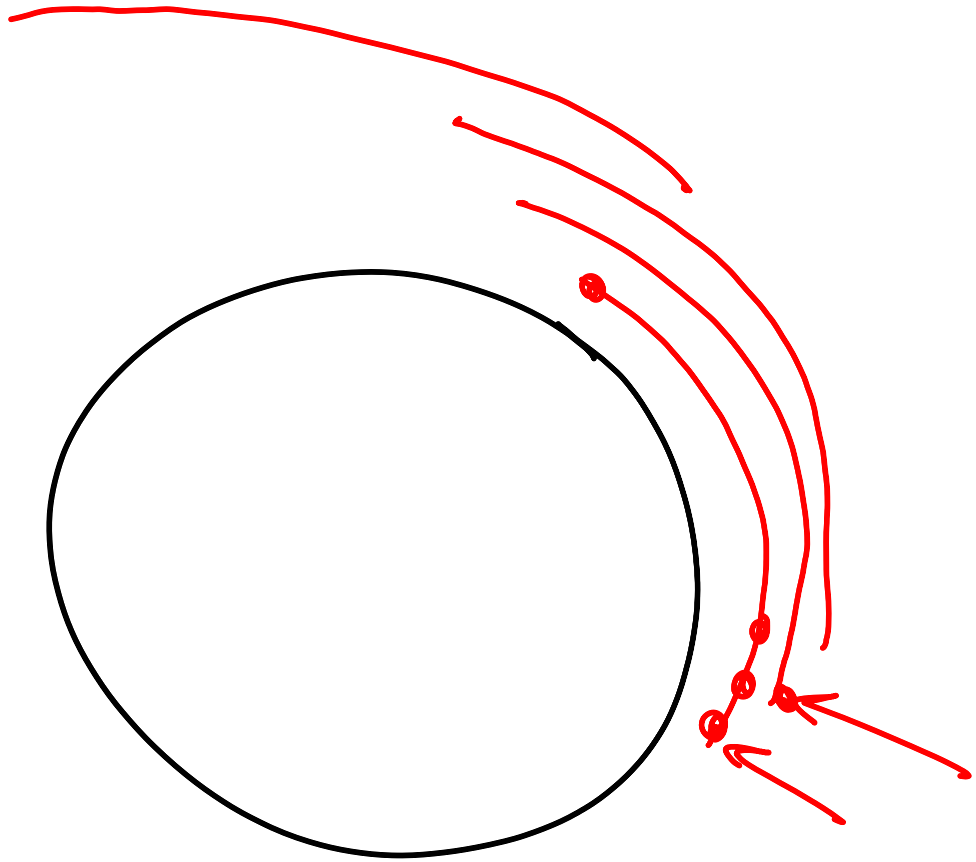
So

$$\sum_{A \in \mathcal{A}} \frac{n\, k!\, (n-k)!}{n!} \leq k$$

$$|\mathcal{A}| \leq \frac{k\, n!}{n\, k!\, (n-k)!} = \binom{n-1}{k-1}$$

$$\Theta(\pi, A) \leq k$$





Pigeon Hole Principle

m objects

n boxes $\sqcup \sqcup \sqcup \cdots \sqcup$

one box gets $\left\lceil \frac{m}{n} \right\rceil$ objects.

Example

Nobody can have more than 10^7 hairs on their body.

In the USA there must be

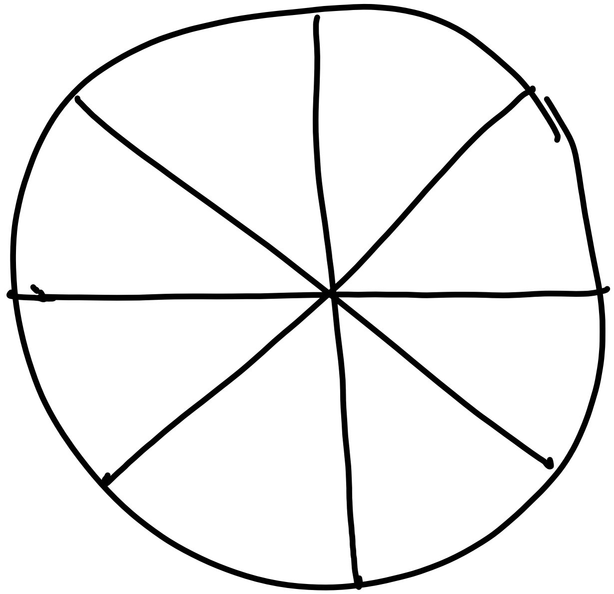
$$\frac{3 \times 10^8}{10^7}$$ people with same number of hairs

$\uparrow$

30

2 Disks

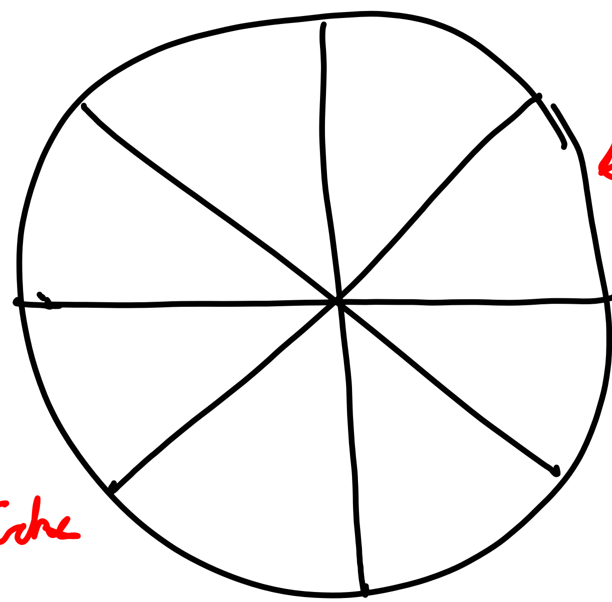
200 sectors



100 Red
100 Blue

Disk 1

Put disk2 on disk1
Match'g corr. sectors have same color



colored arbitrarily

Disk 2

Claim: one can rotate to get ≥ 100 matches

Let q_i = # of matchings s.t. 1 on disk 1 is above i on disk 2.

<u>Claim</u> $\quad q_1 + q_2 + \cdots + q_{200} \geq 200 \times 100$

$\Rightarrow \exists\, i : q_i \geq 100$

How many 1's?

$$
\begin{array}{cc}
 & \text{sectors } j \\
\text{position } i & \left[\begin{array}{c} \\ \underline{1} \text{ if match on sector } j \\ \\ \uparrow \end{array}\right] \leftarrow q_i \text{ 1's} \\
 & 100 \text{ 1's}
\end{array}
$$

(i) By rows

$q_1 + \cdots + q_{200}$

(ii) By columns

Erdös-Szekeres Theorem

Suppose $a_1, a_2, \ldots a_{k^2+1}$ are real numbers.

9, 3, 8, 2, 9, 7, 15, 13, 4, 12

A subsequence: $i_1 < i_2 < \ldots i_s$ is monotone increasing if $a_{i_1} \leq a_{i_2} \leq \ldots \leq a_{i_s}$

Monotone decreasing define similarly

Thm

$\exists$ a monotone subsequence of length $n+1$

Best possible

$n, n-1, \ldots, 1, 2n, 2n-1, \ldots n+1, \ldots, n^2, n^2-1, \ldots n^2-n+1$

n^2 number.

maximum length of a monotone sequence is n