10\25\10

$\mathcal{P}_n$ = power set of $[n]$

= { subsets of $[n]$ }

= $2^{[n]}$

$|\mathcal{P}_n| = 2^n$

Sperner Family

$\mathcal{A} \subseteq \mathcal{P}_n$ is said to be a Sperner family if

$A, B \in \mathcal{A} \Rightarrow A \not\subseteq B$
$A \neq B$ $\quad$ & $B \not\subseteq A$

? How large can a Sperner family be ?

If $\mathcal{A}_k = \{ S \subseteq [n] : |S| = k \}$

is a Sperner family.

$$|\mathcal{A}_k| = \binom{n}{k}$$

Choosing $k = \lfloor \frac{1}{2} n \rfloor$ gives

largest such family

$\exists$ Sperner families of size $\binom{n}{\lfloor \frac{1}{2} n \rfloor}$

Thm

$\mathcal{A} \subseteq \mathcal{P}_n$ is a Sperner family

$\Rightarrow |\mathcal{A}| \leq \binom{n}{\lfloor \frac{1}{2} n \rfloor}$.

Proof

L Y M inequality

if $\mathcal{A}$ is a sperner family the

$$\sum_{A \in \mathcal{A}} \frac{1}{\binom{n}{|A|}} \leq 1 \Rightarrow \sum_{A \in \mathcal{A}} \frac{1}{\binom{n}{\lfloor n/2 \rfloor}} \leq 1$$

$$\Downarrow$$

$$\frac{|\mathcal{A}|}{\binom{n}{\lfloor n/2 \rfloor}} \leq 1$$

Let π be a random permutation of $[n]$.

For $A \in \mathcal{A}$ let

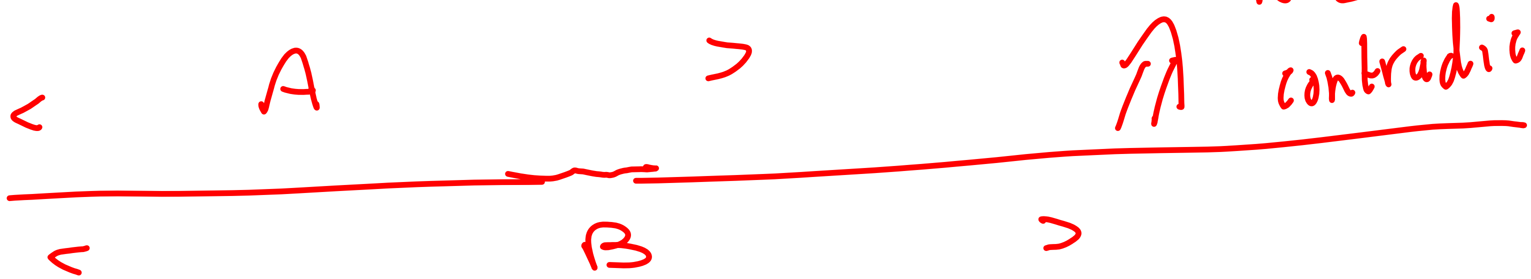
$$\mathcal{E}_A = \{\pi(1), \pi(2), \ldots, \pi(|A|)\} = A$$

(1) $\Pr(\mathcal{E}_A) = \dfrac{1}{\binom{n}{|A|}}$

2) $A \neq B$ & $A, B \in \mathcal{A}$

$$\Longrightarrow \mathcal{E}_A \cap \mathcal{E}_B = \emptyset \qquad \Rightarrow \underline{\underline{\sum_{A \in \mathcal{A}} \Pr(\mathcal{E}_A) \leq 1}}$$

ie can't have both occur



$B \supseteq A$

contradiction

Kraft's Inequality

Let $\chi = \{\underline{x}_1, \underline{x}_2, \cdots \underline{x}_m\}$ be a collection of sequences over some alphabet of size $r = |\Sigma|$.

$\underline{x}_1 =$ aabbcad

$\underline{x}_2 =$ abbbb

$\vdots$

$|\underline{x}_i| = n_i$

length

Suppose $\nexists$ i, j such $i \neq j$

$\underline{x}_i$ is a prefix of $\underline{x}_j$

e.g. $\underline{x}_1 = abc$ & $\underline{x}_2 = abcc$ is not allowed.

Theorem

$$\sum_{\underline{x} \in \mathcal{X}} \frac{1}{r^{|x|}} \leq 1$$

Let $y = y_1 y_2 \cdots\cdots$

be a random string

$\underline{\Psi}_i = y_1 y_2 \cdots y_i$

$$\mathcal{E}_{\underline{x}} = \{ \Psi_{|\underline{x}|} = \underline{x} \}$$

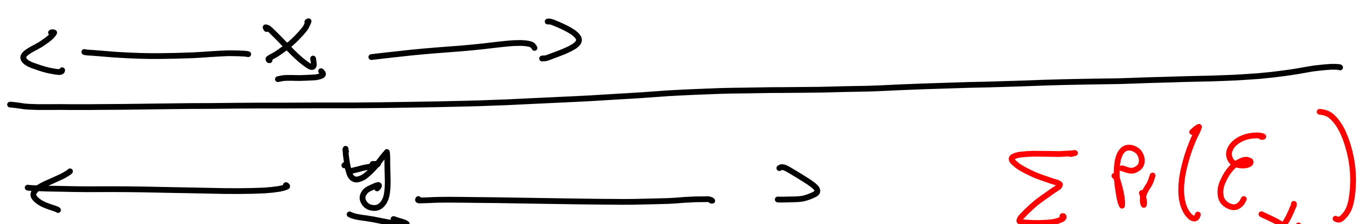
$\underline{x} = acbb$

$$\mathcal{E}_{\underline{x}} = \{ acbb \cdots\cdots\cdots \}$$

(i) $Pr(\mathcal{E}_{\underline{x}}) = \dfrac{1}{r^{|\underline{x}|}}$

(ii) $\underline{x}, \underline{y}$ are in the family then

$$\mathcal{E}_{\underline{x}} \cap \mathcal{E}_{\underline{y}} = \emptyset$$



$$\sum_{\underline{x} \in \mathcal{X}} Pr(\mathcal{E}_{\underline{x}}) \le 1$$

Intersecting Families

A family $\mathcal{A} \subseteq \mathcal{P}_n$ is <u>intersecting</u> if

$$A, B \in \mathcal{A} \Rightarrow A \cap B \neq \emptyset$$

How large can an intersecting family be.

You can't take S & $\bar{S}$ in $\mathcal{A}$

$$\Downarrow$$

$$|\mathcal{A}| \leq 2^{n-1}$$

Take

$$\mathcal{A} = \{ A : 1 \in A \}$$

$$|\mathcal{A}| = 2^{n-1}$$