21-301 Combinatorics
Homework 9
Due: Monday, November 23

1. Consider the following take-away game: There is a pile of n chips. A move consists of removing 3^k chips for some $k \geq 1$. Compute the Sprague-Grundy numbers $g(n)$ for $n \geq 0$.

2. **Poker Nim:** In this game there is a collection of piles of chips plus an extra bag containing a finite number of chips. For a move one can either (i) make a regular Nim move or (ii) take some chips from the bag and put them onto one of the piles. How should one play this game?

3. Consider the following game: There is a pile of chips. A move consists of removing s chips where $s \in S$, assuming that there are at least s chips left. If $|S|$ is finite, show that the Sprague-Grundy numbers satisfy $g(n) \leq |S|$ where n is the number of chips remaining.