

21-301 Combinatorics
Homework 8
Due: Monday, November 16

1. Let \mathcal{A} be an intersecting family of subsets of $[n]$ such that $A \in \mathcal{A}$ implies $k \leq |A| \leq \ell \leq n/2$. Show that

$$|\mathcal{A}| \leq \sum_{i=k}^{\ell} \binom{n-1}{i-1}.$$

2. Let $m = \lfloor n/2 \rfloor$. Describe a family \mathcal{A} of size $2^{n-1} + \binom{n-1}{m-1}$ that has the following property: If $A_1, A_2 \in \mathcal{A}$ are disjoint then $A_1 \cup A_2 = [n]$.
3. Consider the following game: There is a pile of n chips. A move consists of removing any *proper* factor of n chips from the pile. (For the purposes of this question, a proper factor of n , is any factor, including 1, that is strictly less than n). The player to leave a pile with one chip wins. Determine the N and P positions and a winning strategy from an N position.