21-301 Combinatorics
Homework 1
Due: Friday, September 4

1. How many integral solutions of

$$x_1 + x_2 + x_3 + x_4 + x_5 = 100$$

satisfy $x_1 \geq 4$, $x_2 \geq 8$, $x_3 \geq -2$, $x_4 \geq 3$ and $x_5 \geq 0$?

2. Show that if $n \geq q \geq 0$ then

$$\sum_{k=0}^{\ell} \binom{\ell - k}{m}\binom{q+k}{n} = \binom{\ell+q+1}{m+n+1}.$$

3. How many ways are there of placing k 1's and $n - k$ 0's at the vertices of an n vertex polygon, so that every pair of 1's are separated by at least ℓ 0's?