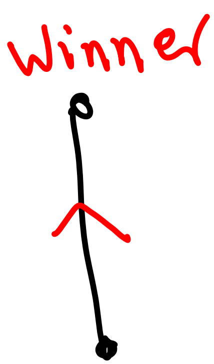
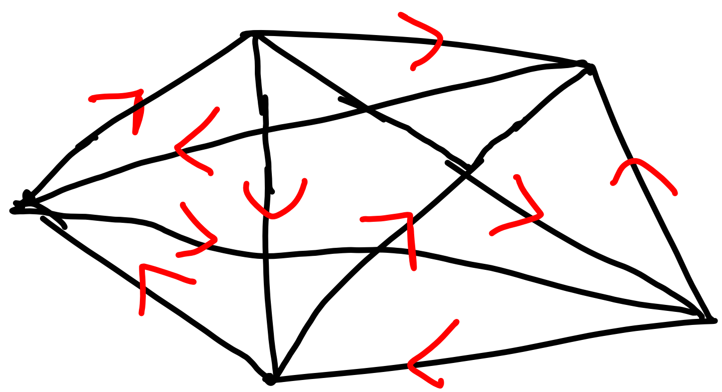
9\28\09

Tournaments

n players.

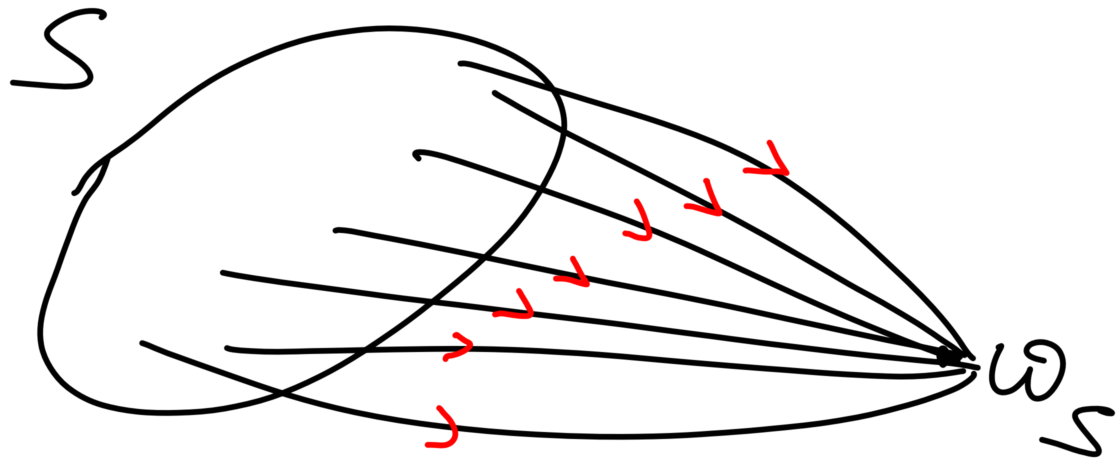
Everybody plays everybody





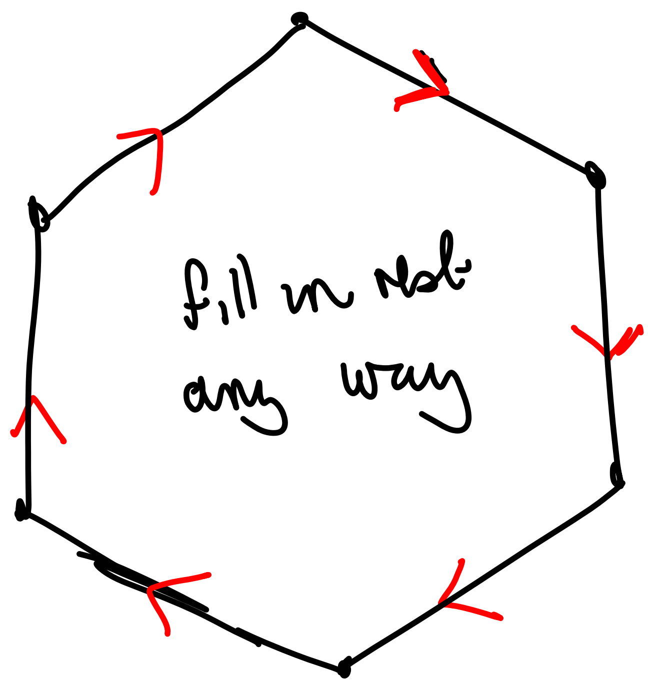
$n=5$

$\forall S : |S| = k$

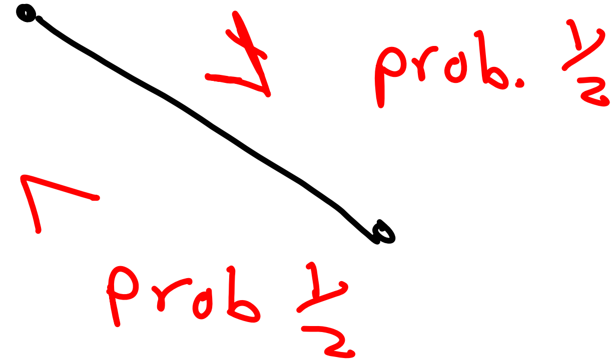


?? Do tournaments exist with this property ??

$k = 1$



Random tournament.



different coin for each edge.

$$\mathcal{E}_S = \{ \nexists \text{ a player that beat everybody in } S \}$$

$$\mathcal{E} = \bigcup_S \mathcal{E}_S$$

We have to show that

$$\Pr(\mathcal{E}) < 1$$

$$\Pr(\mathcal{E}) = \Pr\Big(\bigcup_S \mathcal{E}_S\Big)$$

$$\leq \sum_S \Pr(\mathcal{E}_S) = \left(1 - \frac{1}{2^k}\right)^{n-k}$$

$\Pr(\mathcal{E}_S) = ??$

- $v \notin S$

$|S| = k$

$\Pr[v \text{ beats } S] = \frac{1}{2^k}$

$\Pr[v \neg \text{beats } S] = 1 - \frac{1}{2^k}$

$$P(\mathcal{E}) \leq \sum_{|S|=k} \left(1-\frac{1}{2^k}\right)^{n-k}$$

$$= \binom{n}{k}\left(1-\frac{1}{2^k}\right)^{n-k}$$

k is fixed and n is large

$$\leq n^k \left(1-\frac{1}{2^k}\right)^{n-k}$$

$$\leq n^k e^{-(n-k)/2^k}$$

$1+x \leq e^x$, $\forall$ real x

$$\Pr(\mathcal{E}) \leq n^k e^{-(n-k)/2^k}$$

$$\leq n^k e^{-n/2^{k+1}}$$

if $n \geq 2k$.

$$\log(\text{RSH}) = k \log n - n/2^{k+1}$$

choose n large enough so that

$$\frac{n}{\log n} > k 2^{k+1}$$

$$1 + x \leq e^{x}$$

(i) $x \geq 0$

$$1 + x \leq 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

(ii) $x \leq -1$

$$1 + x \leq 0 \leq e^{x}$$

(iii) $-1 < x < 0$; $x = -y$ $\quad 0 < y < 1$

$$e^{-y} - 1 + y = \left(\frac{y^2}{2!} - \frac{y^3}{3!}\right) + \left(\frac{y^4}{4!} - \frac{y^5}{5!}\right) + \cdots$$

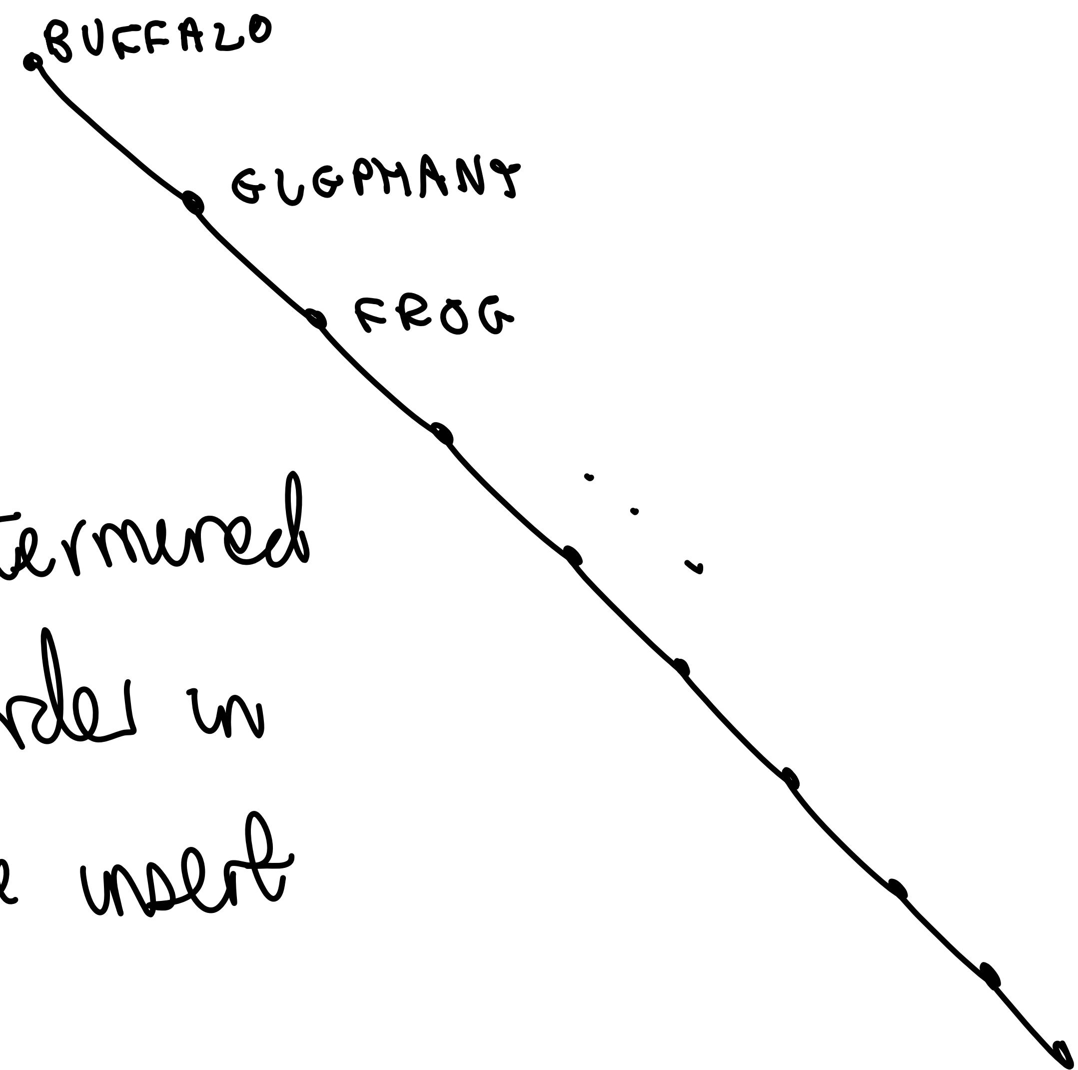
$$\geq 0 \qquad\qquad \geq 0$$

Random Binary Search Trees

Data structure

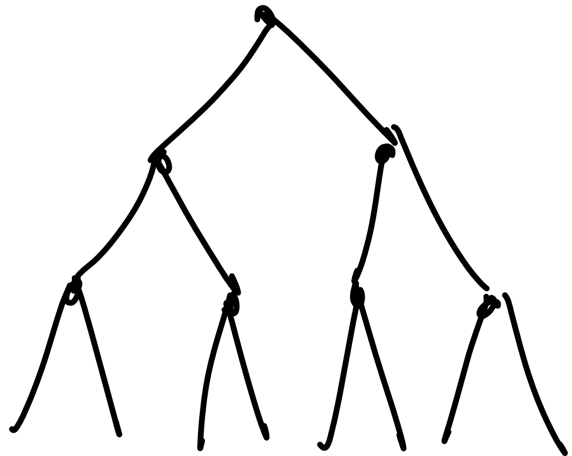
Depth/Height of tree determines maximum time to find/insert name.

BAD Tree



Tree is determined by the order in which we insert things

PERFECT TREE



Perfectly Balanced.

$$\text{Height} = O(\log n)$$

#items

Random model of construction

Random sequence of coin flips .

R R L R L L R L R R L - -