9\16\09

Unkown sequence a_n, $n = 0, 1, 2, \ldots$

a_n = number of ....

It may be possible to find a recurrence relation:

e.g. $a_0 = 1$ & $a_n = 2a_{n-1}^2$

a_n = function of

$a_0, a_1, \ldots, a_{n-1}$

This determines the sequence and we can compute any a_n for fixed n.

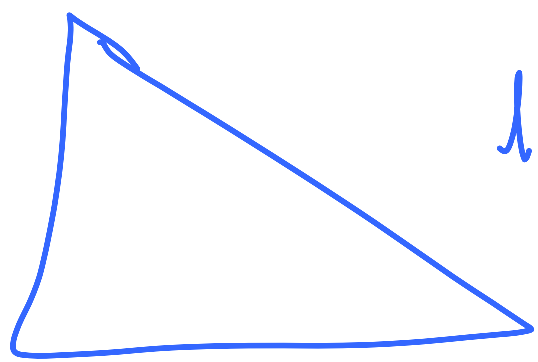
$a_0 = 2$ $\quad a_1 = 4 \quad a_2 = 16$

$a_n = a_{n-1}^2 \quad a_3 = 256 \quad a_4 = 65536$

Goal is to use the recurrence relation to obtain a formula for a_n.

Sequence $\equiv$ ∞ # of variables

Recurrence $\equiv$ ∞ # equations



lower triangular, solvable

Example

$$a_n = a_{n-1} + a_{n-2}, \quad n \geq 2$$

$$a_0 = a_1 = 1$$

$$b_n = |B_n|$$

$$B_n = \{ x \in \{a, b, c\}^n : \text{aa does not occur in } x \}$$

$b_0 = 1$;

$b_1 = 3$; $b_2 = 8$

$$b_n = 2 b_{n-1} + 2 b_{n-2} \quad , n \geq 2$$

$$b_n = 2b_{n-1} + 2b_{n-2}$$

$$B_n = \underset{b_{n-1}}{B_n^{(b)}} \cup \underset{b_{n-1}}{B_n^{(b)}} \cup \underset{2b_{n-2}}{B_n^{(a)}}$$

$$|B_n^{(b)}| = b_{n-1}$$

$x \in$

$$b\underbrace{* * * - - - \cdots *}_{\text{any member of } B_{n-1}}$$

$$|B_n^{(b)}| = b_{n-1} = |B_n^{(a)}|$$

$B_n^{(a)}$

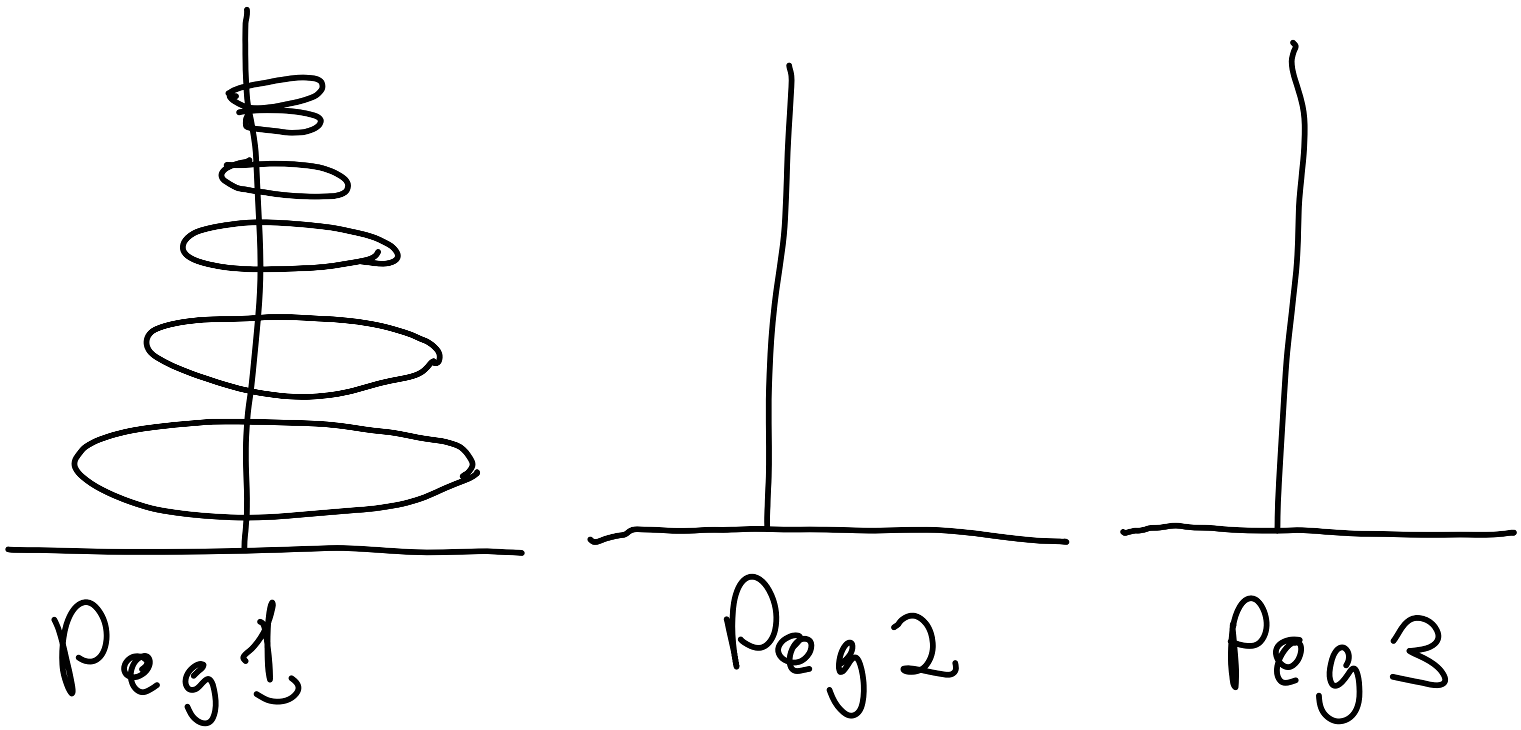
$x \in \uparrow$

$x = a\, b * * \cdots$

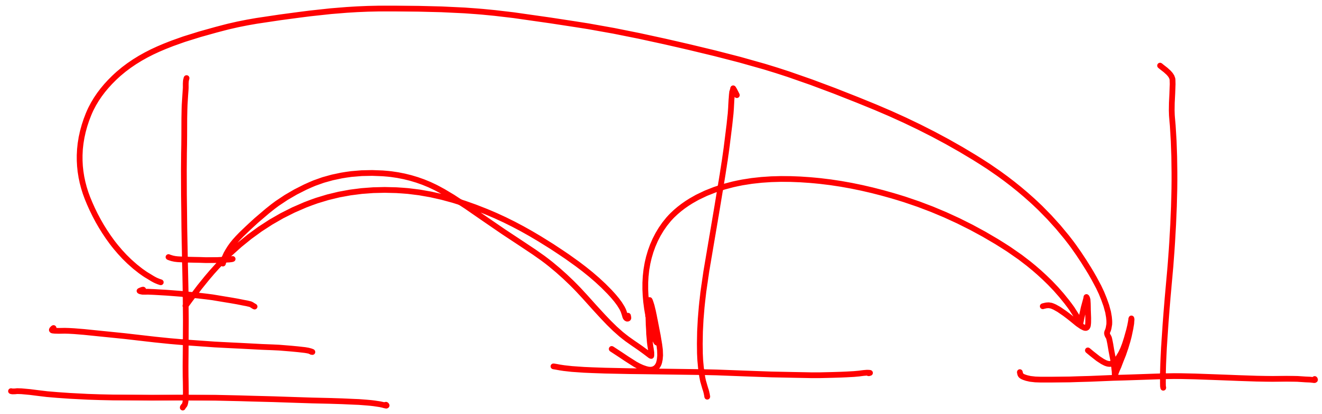
$a\, c\, \underbrace{* \cdots\cdots}_{\text{any member of } B_{n-2}}$

$$|B_n^{(a)}| = 2 b_{n-2}$$

Towers of Hanoi

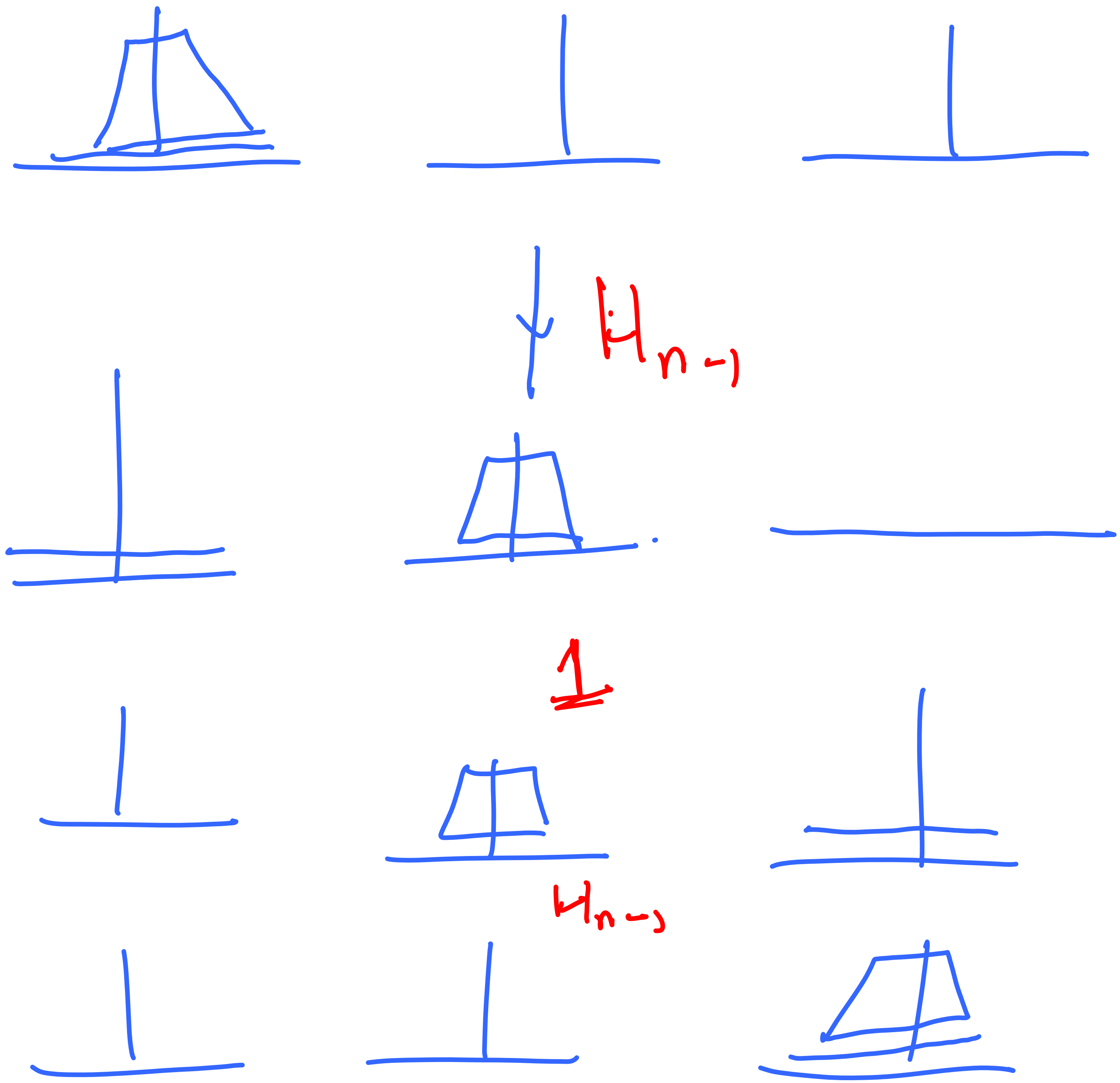


Golden Rings of different sizes: Rule smaller onto large



H_n = minimum number of moves

⋮


H_{n-1}
1
H_{n-1}

$$H_n = 2H_{n-1} + 1$$

$$H_1 = 1$$

Question: How many are need with 4 pegs.

Generating Functions

$a_0, a_1, a_2, \dots a_n, \dots$

our unknown

$$a(x) = a_0 + a_1 x + a_2 x^2 + \dots + a_n x^n + \dots$$

1-1 correspondence between

Sequences & power series

$a_0, a_1, \dots a_n, \dots$

$a(x) = a_0 + a_1 x + a_2 x^2 + \dots$

Goal becomes:

compute $a(x)$

Recurrence

$\downarrow$

equation in $a(x)$

+ solve for $a(x)$

$\searrow$ extract coefficients