8\31\09

$$|\text{PATHS}_{>}(a,b)| =$$

$$|\text{PATHS}[(0,1) \to (a,b)]| - |\text{PATHS}[(1,0) \to (a,b)]|$$

$$\binom{a+b-1}{a} - \binom{a+b-1}{a-1}$$

$$\downarrow$$

$$\binom{a+b}{a} \cdot \frac{b}{a+b} - \binom{a+b}{a} \frac{a}{a+b}$$

$$= \frac{b-a}{a+b} \binom{a+b}{a}$$

$$\left| \mathrm{PATHS}_{\geq} \left((a, b) \right) \right|$$

$$\parallel$$

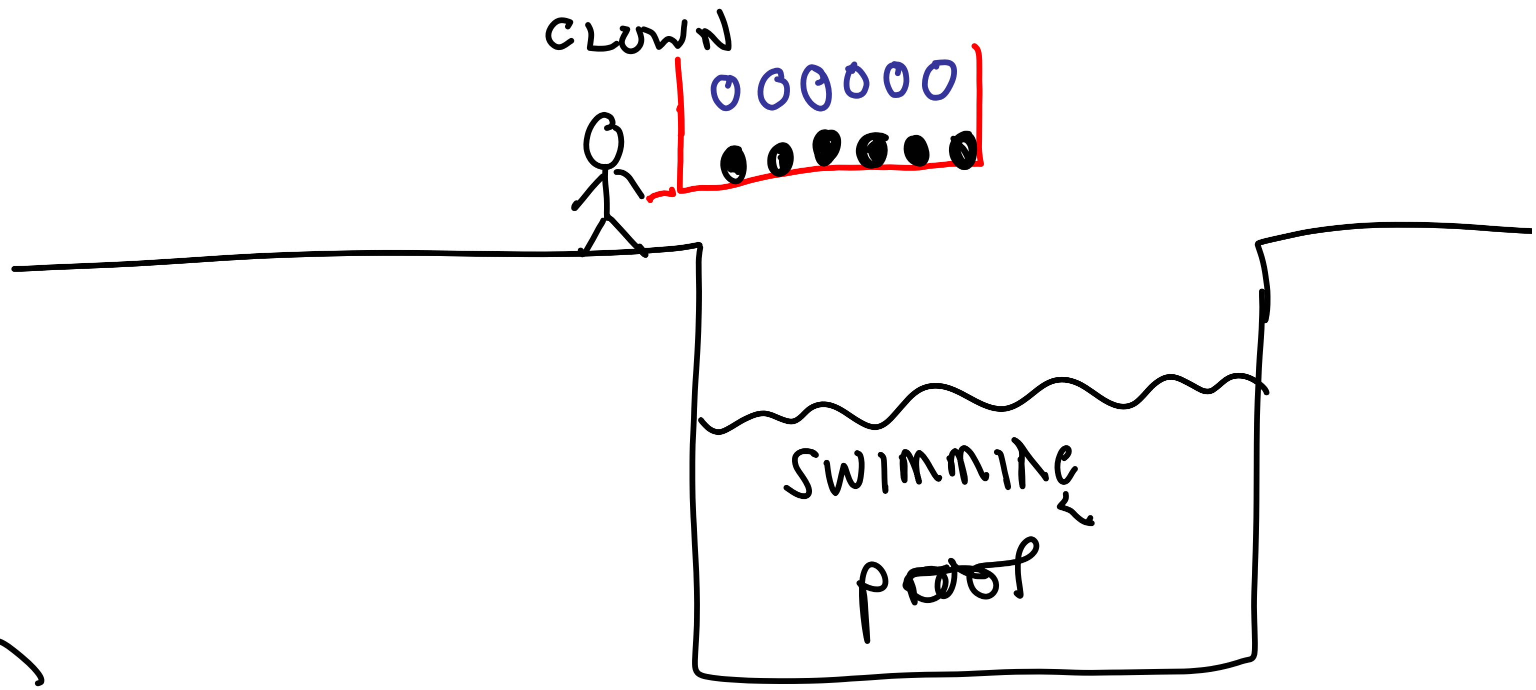
$$\left| \mathrm{PATHS}_{>} (a, b+1) \right|$$

$$\left| \mathrm{PATHS}_{\geq} (a, b) \right| = \frac{b-a+1}{a+b+1} \binom{a+b+1}{a}$$

Ex: $a = b$

$$\frac{1}{a+1} \binom{2a}{a}$$

CATALAN NUMBER



~~6~~ n BLACK BALLS

n WHITE BALLS

At each step: Pick a ball, throw away.

Black: ← U

White: → R

What is probability the clown stays dry.

ANSWER $\frac{1}{n+1}$

Multi-Sets

$\{a, a, b, b, c, d, d, d\}$

$\{2 \times a, 2 \times b, 1 \times c, 3 \times d\}$

Question: How many permutation of $\{a_1 \times 1, a_2 \times 2, \ldots, a_n \times n\}$ are there?

$A = \{a, a, b, b\}$

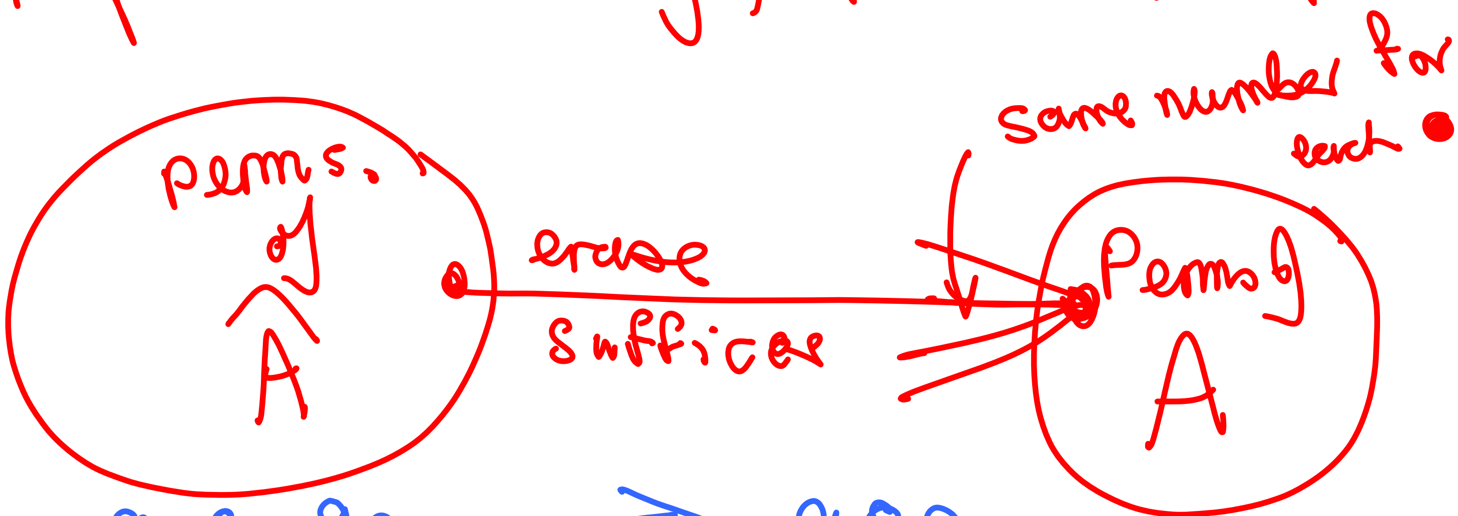
$$\left.\begin{array}{cccc} a & a & b & b \\ a & b & a & b \\ a & b & b & a \\ b & b & a & a \\ b & a & b & a \\ b & a & a & b \end{array}\right\} 6$$

$A = \{3 \times a,\ 4 \times b,\ 5 \times c,\ 2 \times d,\ 4 \times e\}$

$\hat{A} = \{a_1, a_2, a_3, b_1, b_2, b_3, b_4, \ldots, e_4\}$

$|\hat{A}| = 18$

permutations of $\hat{A}$ is $18!$



$a_1 e_4 a_3 \cdots \Longrightarrow aea \ldots$

e a b b c d a c d b e e a c e b c e

Put suffices back

a	a	a
1	2	3
1	3	2
2	1	3
2	3	1
3	1	2
3	2	1

permutations of

$$\frac{18!}{3!\,4!\,5!\,2!\,4!}$$

permutations of $\{a_1 \times 1, a_2 \times 2, \ldots, a_n \times n\}$

$$= \frac{m!}{a_1!\,a_2!\cdots a_n!} = \binom{m}{a_1, a_2, \ldots, a_n}$$

$m = a_1 + \cdots + a_n$

multinomial coefficient

$$(x_1 + x_2 + \cdots + x_n)^m =$$

$$\sum_{\substack{a_1 + \cdots + a_n = m \\ a_1 \geq 0, \ldots, a_n \geq 0}} \binom{m}{a_1, a_2, \ldots, a_n} x_1^{a_1} x_2^{a_2} \cdots x_n^{a_n}$$

permutations of $\{a_1 \times x_1, a_2 \times x_2, \ldots, a_n \times x_n\}$

$$(\quad)(\quad)\cdots(\quad)(\quad)$$

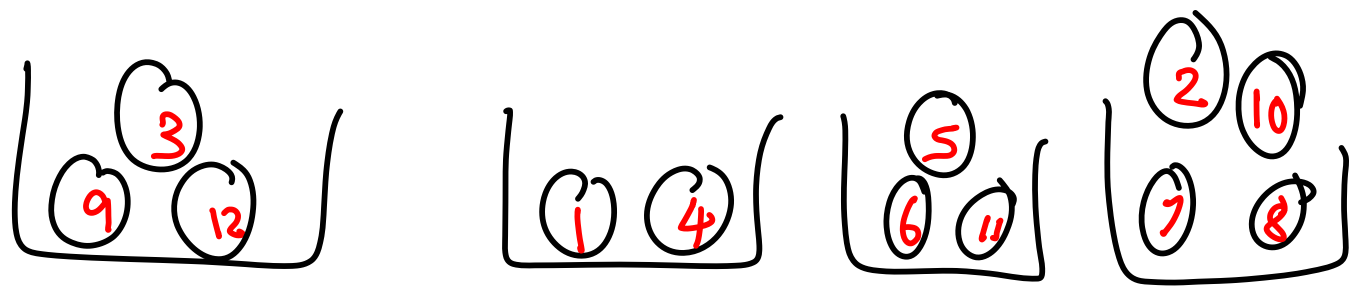
$$1 \qquad 2 \qquad \cdots \qquad m-1 \qquad m$$

Balls in boxes

m distinguishable balls

n boxes.

ways of placing balls so that Box i gets b_i balls is $\binom{m}{b_1, \dots b_n}$



boxes

$\Updownarrow$ permutation

2 4 1 2 3 3 4 4 1 4 3 1.

Permutation?

$\{1 \times 3,\ 2 \times 2,\ 3 \times 3,\ 4 \times 4\}$