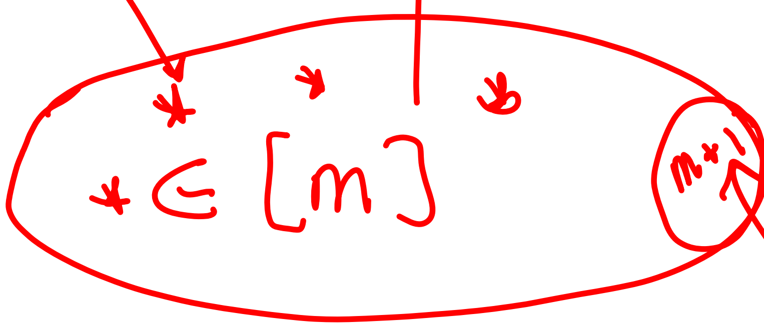
8/28/09

$$\binom{k}{k} + \binom{k+1}{k} + \cdots \binom{m}{k} \cdots + \binom{n}{k} = \binom{n+1}{k+1}$$

$|S_m|$ $\quad$ $|S_n|$

of $(k+1)$-subsets of $n+1$

k of them



largest element

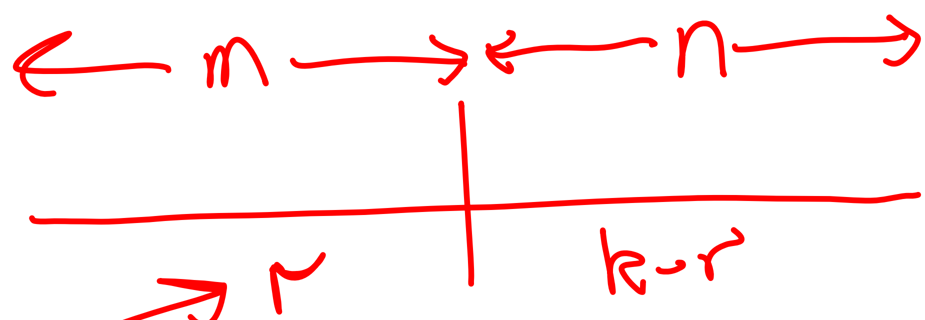
$k+1$ - set

$$S_m = \{ X : |X| = k+1 \ \& \ \max X = m+1 \}$$

Vandermonde's Identity

$$\sum_{r=0}^{k} \binom{m}{r}\binom{n}{k-r} = \binom{m+n}{k}$$

# of subsets of $[m+n]$ of size k



decide how many to chose from $0 \le r \le k$

Binomial Theorem

$$(1+x)^n = \sum_{r=0}^{n} \binom{n}{r} x^r$$

$(1+x)(1+x) \cdots (1+x)$

choose x r lines

1 $n-r$ line

as we multiply out

$\underline{x = 1}$

$$(1+1)^n = \binom{n}{0} + \binom{n}{1} + \cdots + \binom{n}{n}$$

\# subsets of $[n]$

$\underline{x = -1}$

$$\binom{n}{0} - \binom{n}{1} + \binom{n}{2} + \cdots = (1-1)^n = 0$$

$$\binom{n}{0} + \binom{n}{2} + \cdots = \binom{n}{1} + \binom{n}{3} + \cdots$$

\# even subsets = \# odd subsets

$$i = \sqrt{-1}$$

$$(1+i)^n = \sum_{r=0}^{n} \binom{n}{r} i^r$$

$$A + Bi = \binom{n}{0} + \binom{n}{1} i + \binom{n}{2} i^2 + \binom{n}{3} i^3 + \cdots$$

$$\binom{n}{0}$$

$$\binom{n}{1} i$$

$$-\binom{n}{2}$$

$$-\binom{n}{3} i$$

$$\frac{1+i}{\sqrt{2}} = \cos\frac{\pi}{4} + i\sin\frac{\pi}{4} = e^{i\pi/4}$$

$$\vdots$$

So

$$(1+i) = \sqrt{2}\, e^{i\pi/4}$$

and

$$(1+i)^n = 2^{n/2} e^{in\pi/4}$$

$$= 2^{n/2}\left[\cos n\pi/4 + i \sin n\pi/4\right].$$

Thus

$$\binom{n}{0} - \binom{n}{2} + \binom{n}{4} - \cdots = 2^{n/2} \cos n\pi/4$$

$$\binom{n}{1} - \binom{n}{3} + \binom{n}{5} - \cdots = 2^{n/2} \sin n\pi/4.$$

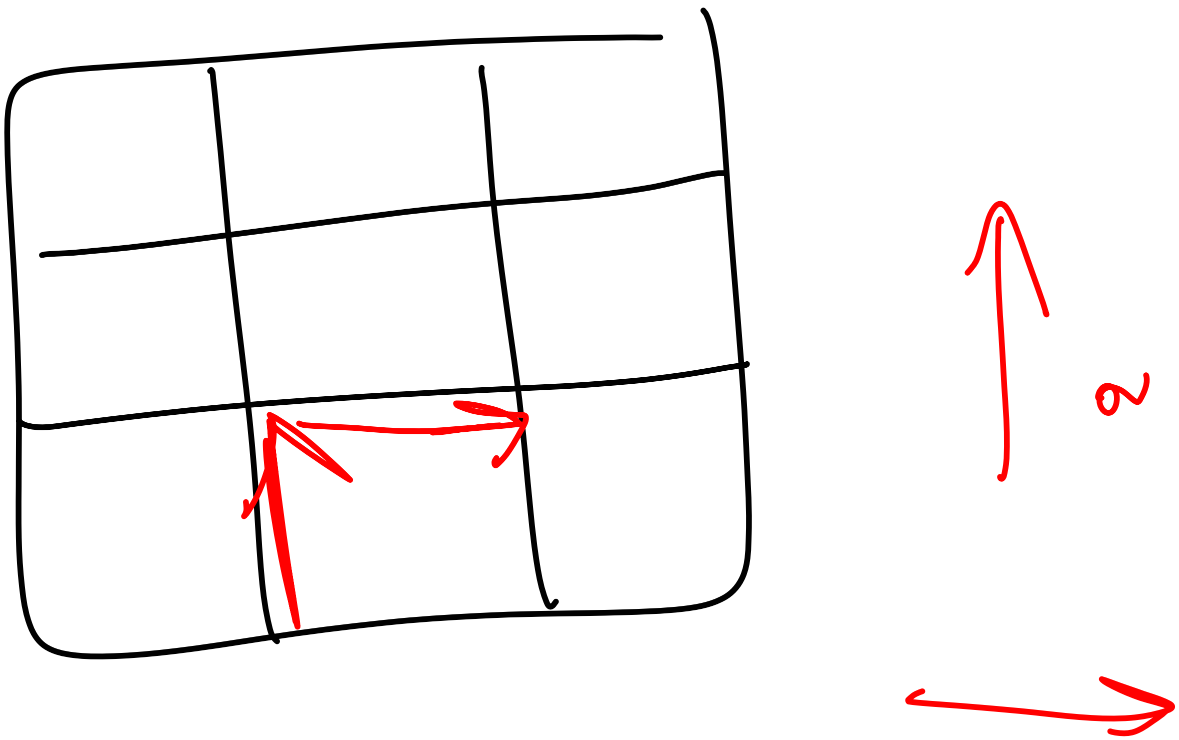
$$\sum_{k=0}^{n} k\binom{n}{k} = n\,2^{n-1}$$

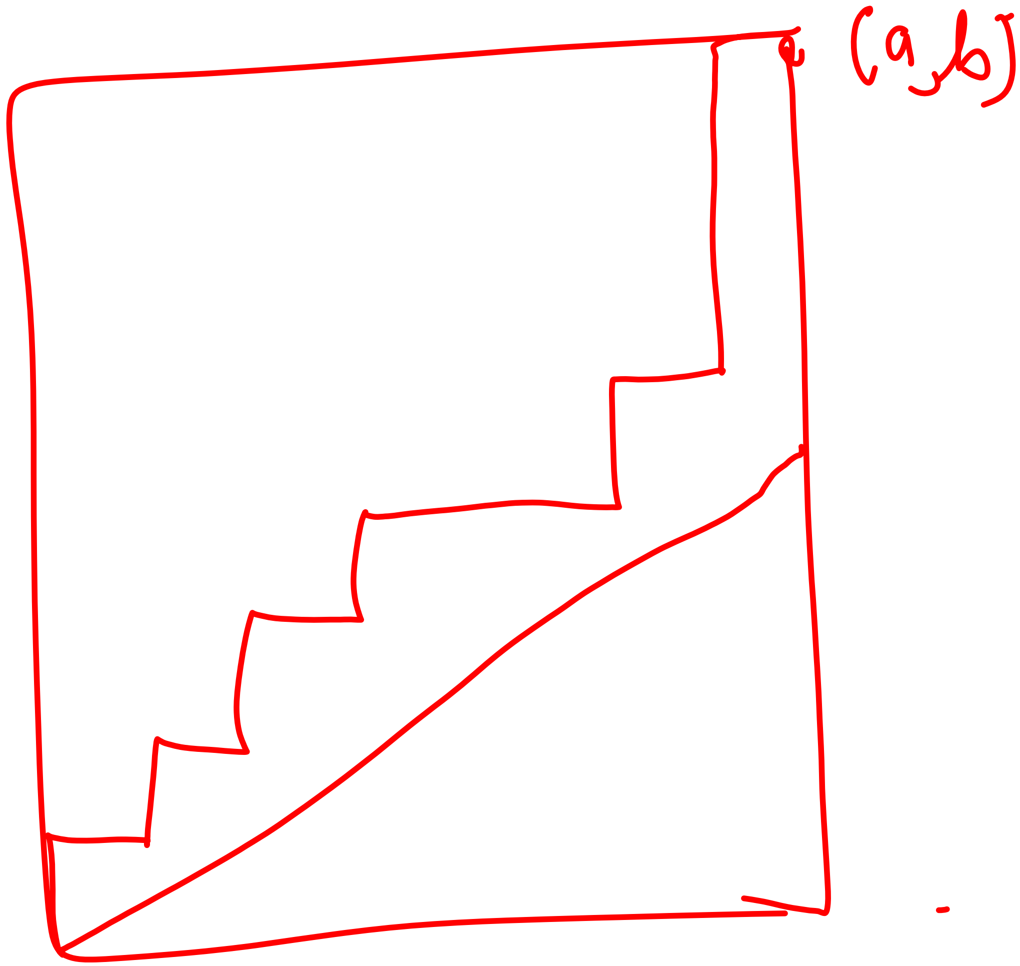
Differentiate

$$n(1+x)^{n-1} = \sum_{k=0}^{n} k\binom{n}{k}x^{k-1}$$

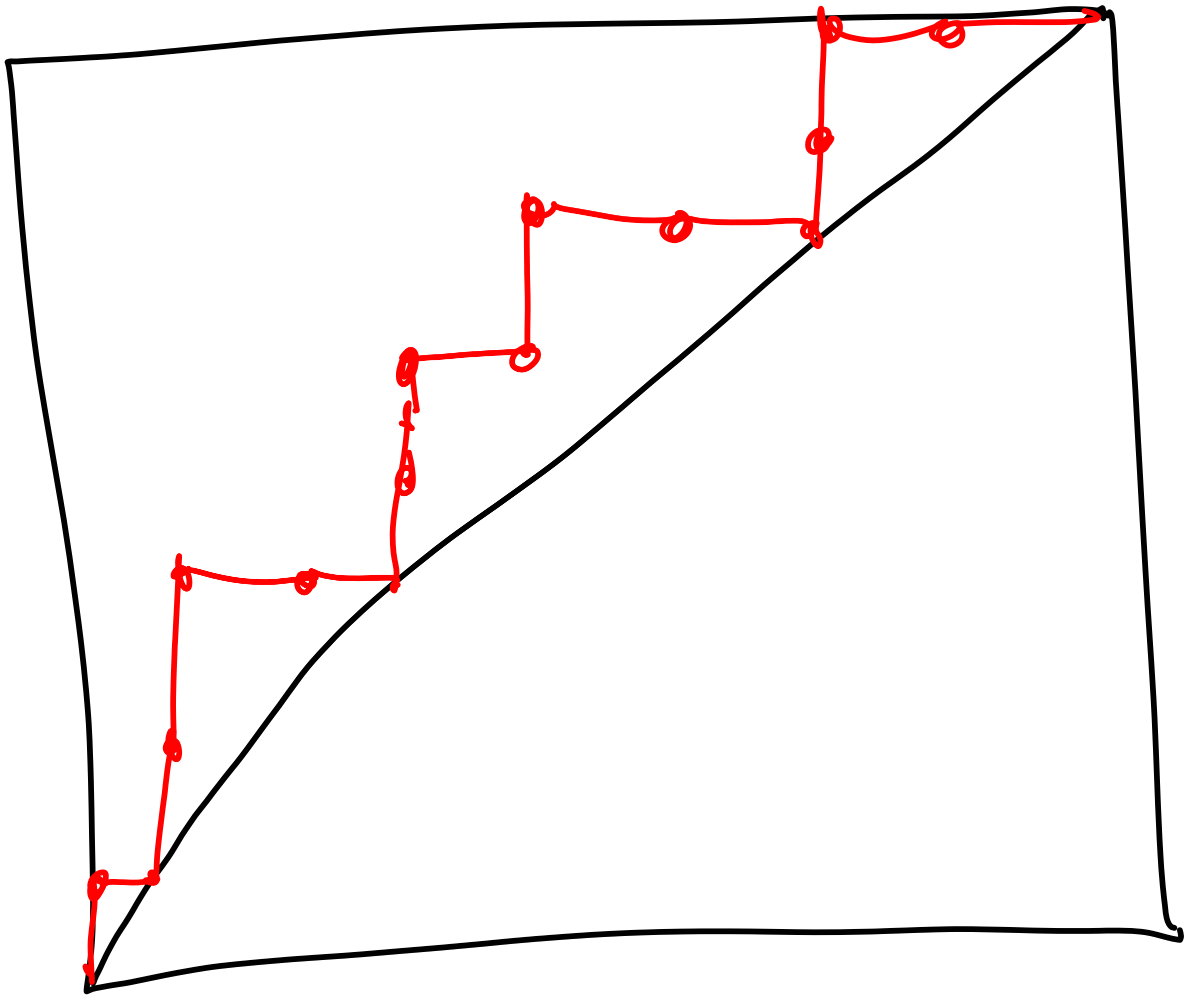
Put $x = 1$

Monotone Paths





PATHS $>$ (a, b)



$\text{PATHS}_{\geq}(a, b)$

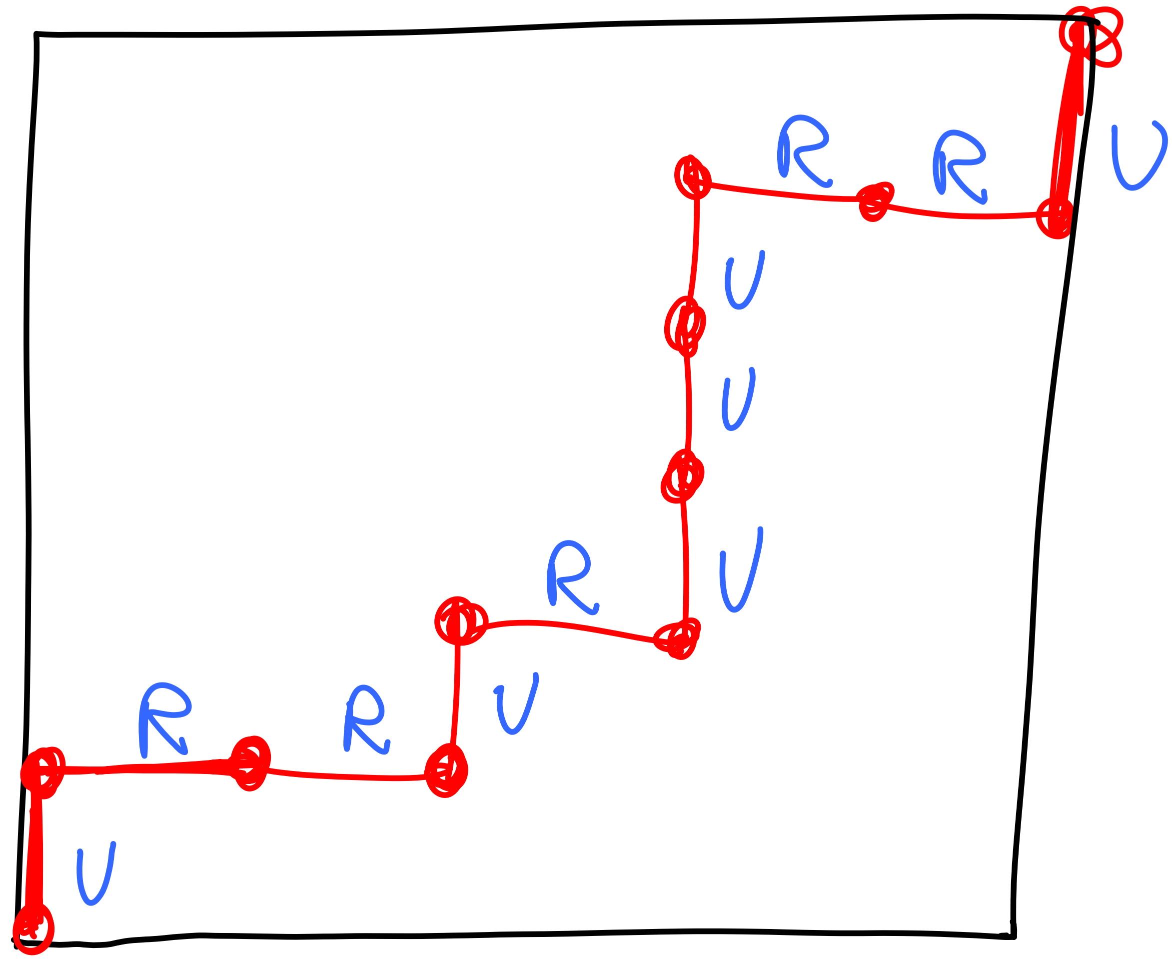
PATHS(a,b)

size $\binom{a+b}{b}$

bijection

strings of a R's
b U's

size $= \binom{a+b}{a}$



PATH $\longrightarrow$ URRURUUURRU

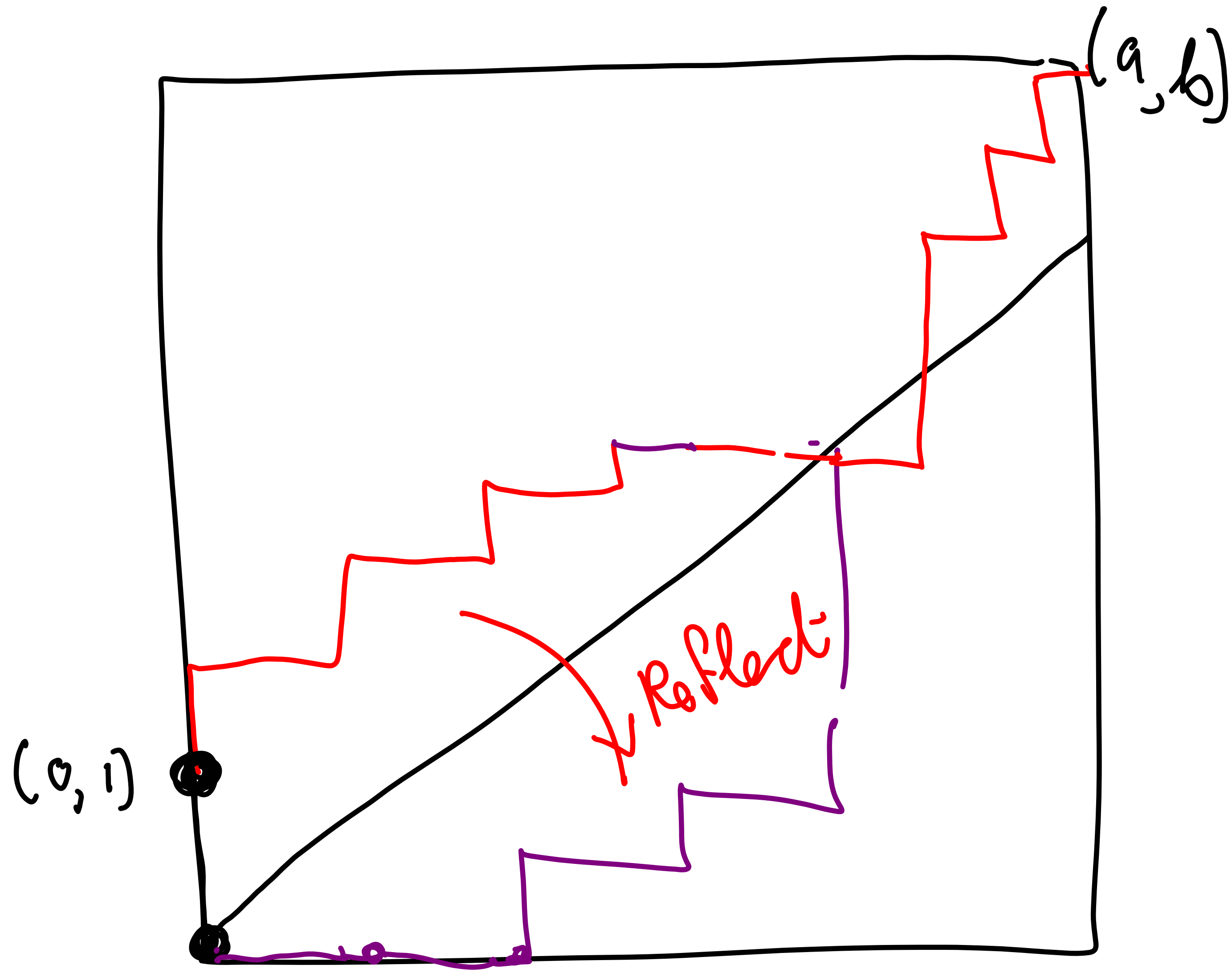
$|\text{PATHS}_{>}(a,b)|$

first move

$$|\text{PATHS}_{>}(a,b)| = |\text{PATHS}((0,1)\to(a,b))|$$

$$- |\text{PATHS}_{\not>}((0,1)\to(a,b))|$$

$$= \binom{a+b-1}{a} -$$



Reflection is a path from (1, 0) to (a, b)

$$\left|\mathrm{PATHS}_{\not>}\big((0,1) \to (a,b)\big)\right|$$

$$= \left|\mathrm{PATHS}\big((1,0) \longrightarrow (a,b)\big)\right|$$

$$= \binom{a+b-1}{a-1}$$