8/24/09

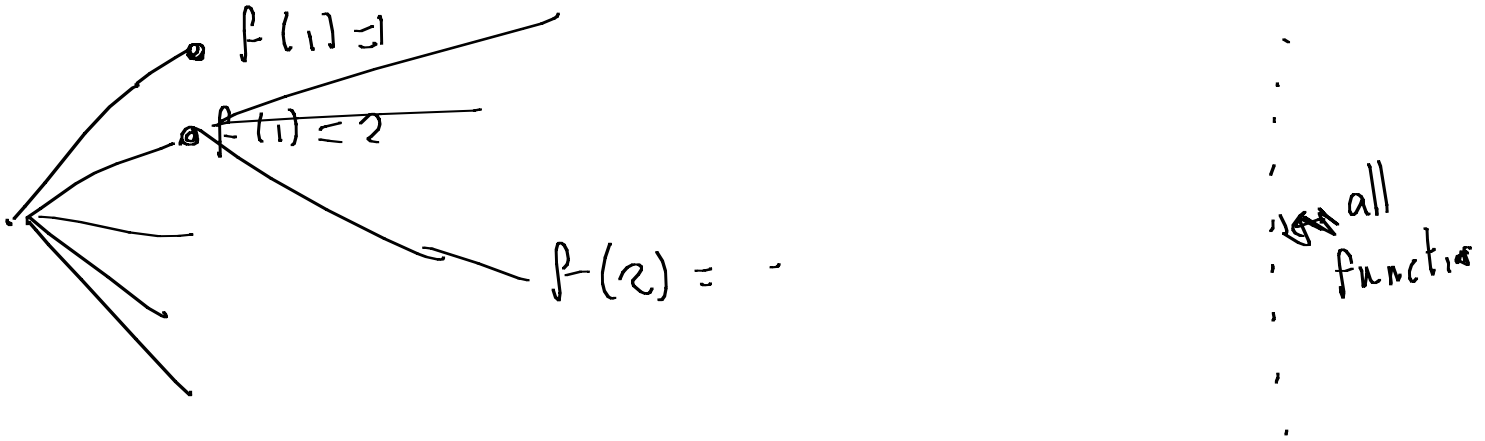
$\{1, 2, \cdots n\}$

How many functions $f : [n] \to [m]$

$\phi(m,n) = m^n$

Choose $f(1)$ & $f(2)$ & $\cdots$ & $f(n)$

m choices $\times$ m choices $\times \cdots \times m$



Notice:

$$\#\text{functions} = \#\text{sequences}$$

$$\begin{array}{ccccc} a_1, & a_2, & \cdots, & a_n \\ \| & \| & & \| \\ f(1) & f(2) & & f(n) \end{array}$$

$$\#\text{sequences} = m^n$$

Let $\psi(n)$ = # of subsets of $[n]$.

$$\psi(n) = 2^n$$

Proof

(1) Induction

Base Case

$n = 0$, $\psi(0) = 1$

Assume true for n ie $\psi(n) = 2^n$

$$\psi(n+1) = \psi(n) + \psi(n)$$

$\psi(n)$: contains $n+1$

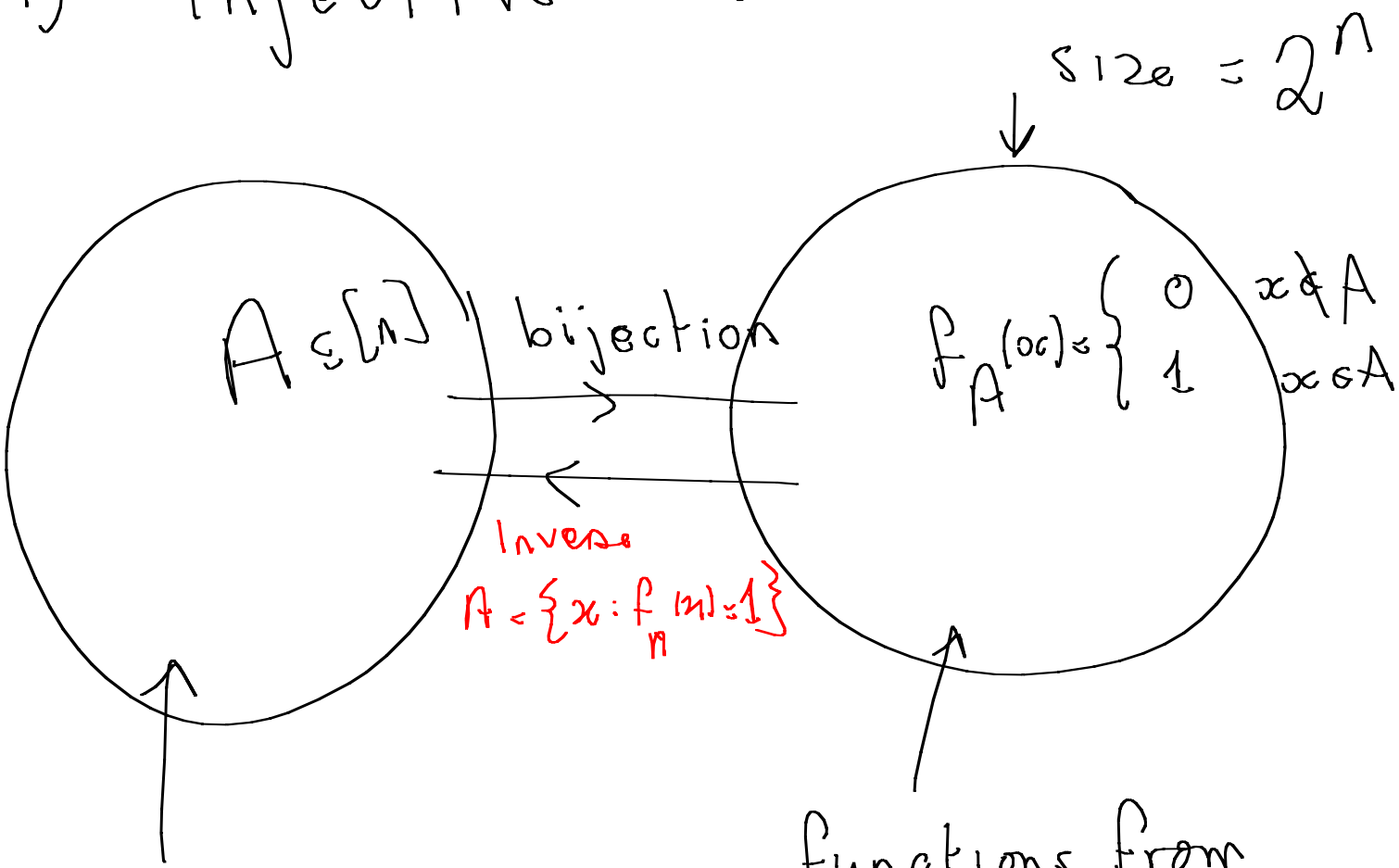
$\psi(n)$: not contains $n+1$

$(n+1)$ plus a subset of $[n]$

$$= 2\psi(n)$$

$$= 2^{n+1}.$$

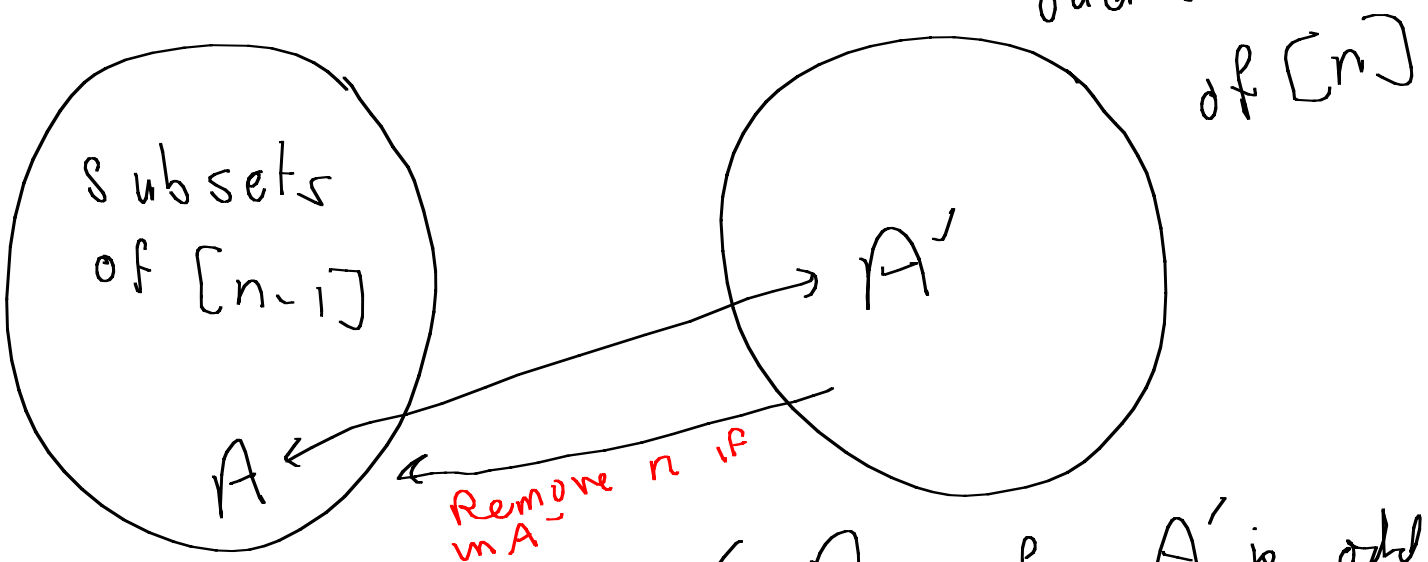
(11) Injective Proof



2 finite sets A, B. $\Rightarrow |A| = |B|$
Bijection $A \to B$

$\psi_{odd}(n) = \#$ odd cardinality subsets of $[n]$.

$$\psi_{odd}(n) = 2^{n-1}$$



$$A' = \begin{cases} A & \text{if } A' \text{ is odd} \\ A \cup \{n\} & \text{if } A' \text{ is even} \end{cases}$$