11\11\09

Erdős-Ko-Rado theorem

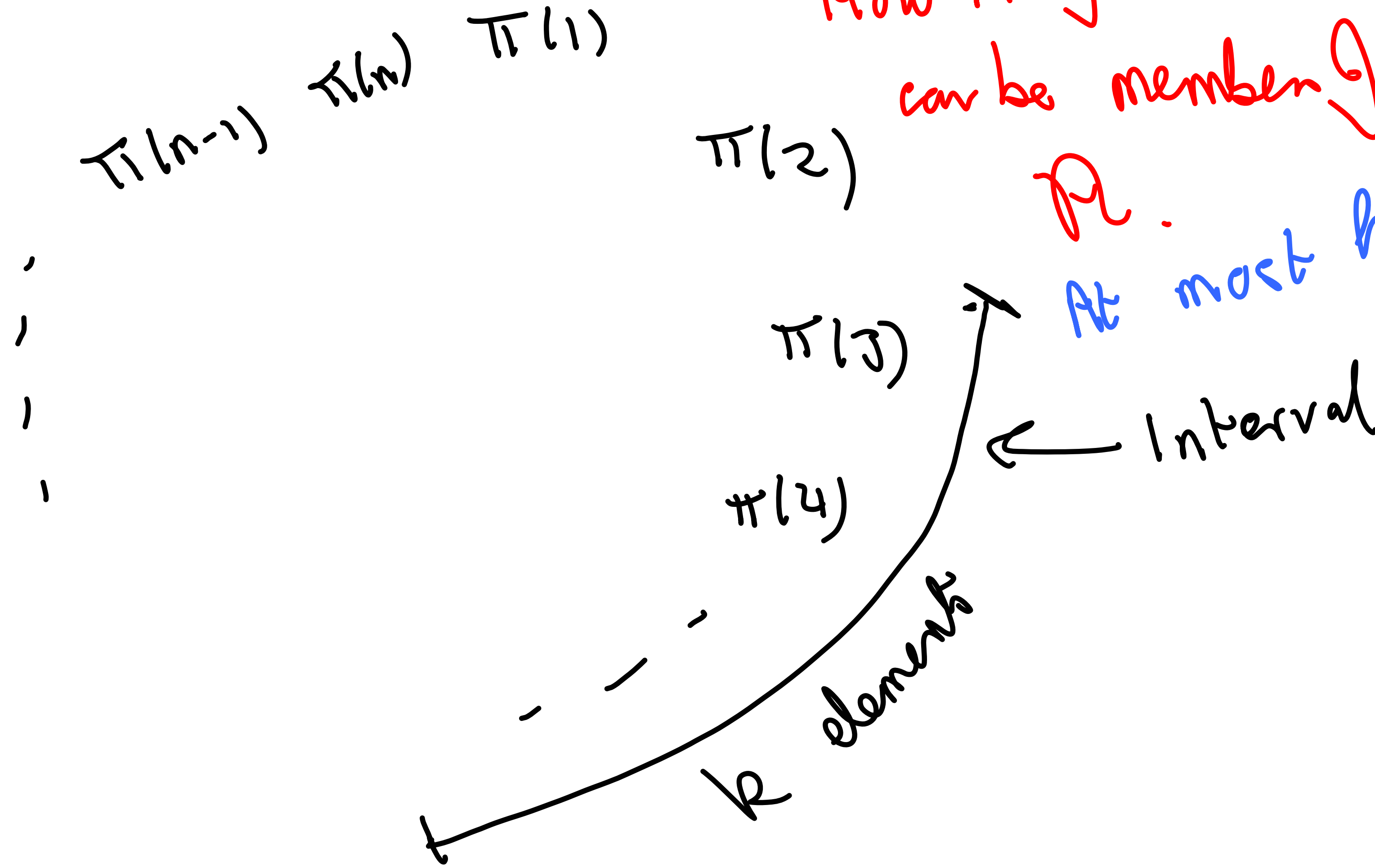
$\mathcal{A}$ is an intersecting family

$A \in \mathcal{A} \Rightarrow |A| = k \leq \lfloor n/2 \rfloor$

$$|\mathcal{A}| \leq \binom{n-1}{k-1}$$

Upper bound is achieved by all k-sets containg element 1.

π is a permutation of $\{1, 2, \ldots, n\}$



How many intervals can be member of R.

At most k

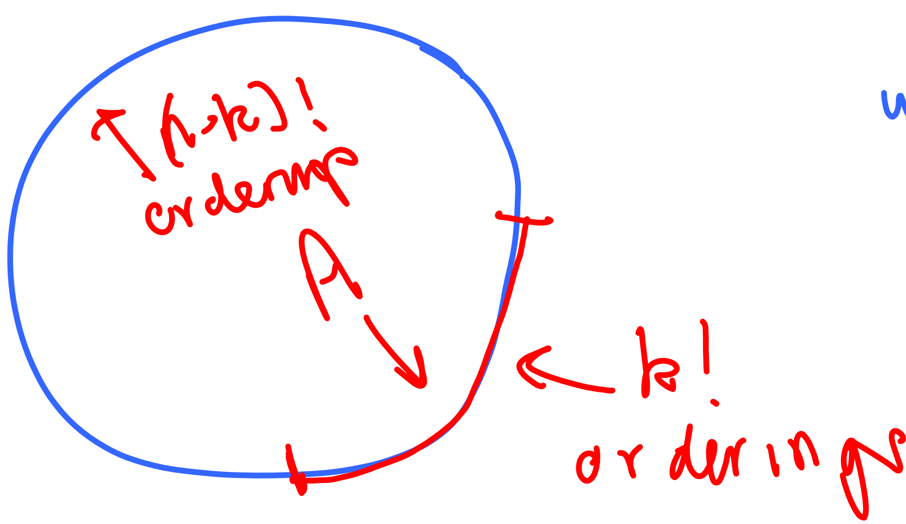
Claim: $\forall \pi, \sum_{A \in \mathcal{A}} \mathbb{1}_{A \text{ is an interval}} \le k$

Let π be a random permutation

$$\Pr(A \text{ is an interval}) = n \frac{k!\,(n-k)!}{n!}$$

where to start

permutation that place A as the interval



$$k \geq E\left(\sum_{A \in \mathcal{A}} 1_{A \text{ is interval}}\right) = \sum_{A \in \mathcal{A}} n \frac{k!\,(n-k)!}{n!}$$

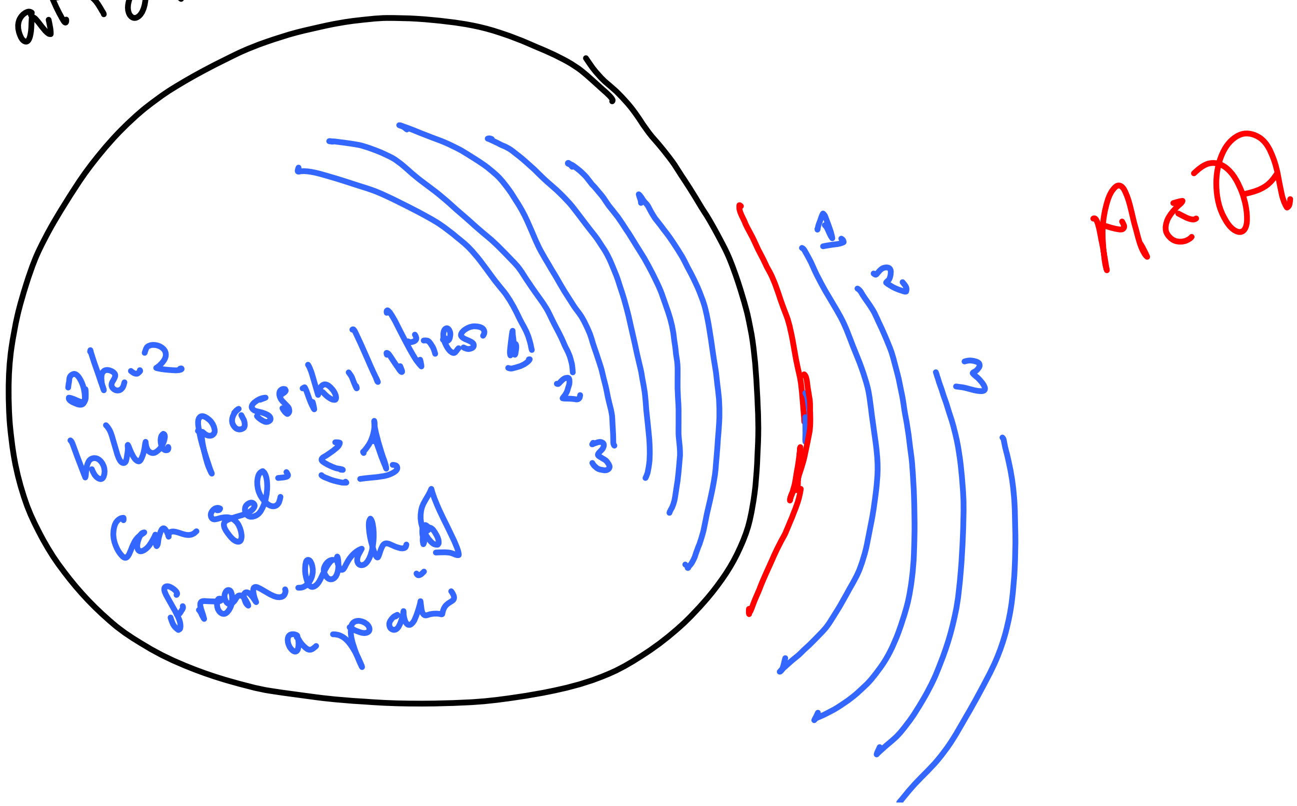
$$= \frac{k}{\binom{n-1}{k-1}} \cdot |\mathcal{A}|$$

$$\Rightarrow \quad |\mathcal{A}| \leq \binom{n-1}{k-1}$$

To finish

Fix π: show $\leq k$ intervals from $\mathcal{A}$.

permutation

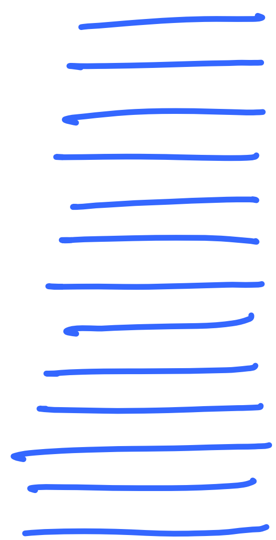


Combinatorial Games

2 player, perfect information games.

Game 1

Player A, B



Move: Remove 1, 2, 3 or 4 chips.

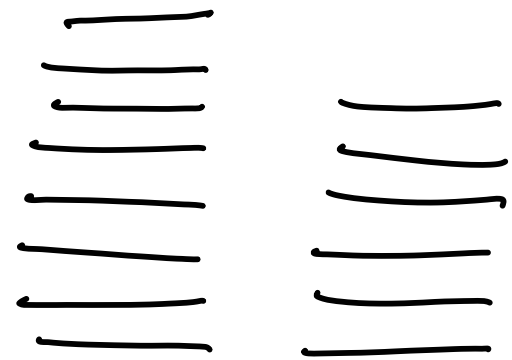
Winner: Person that takes last chip.

A goes first

# chips	Winner
1	A
2	A
3	A
4	A
5	B
6	A
7	A
8	A
9	A
10	B
11	A
12	A
13	A

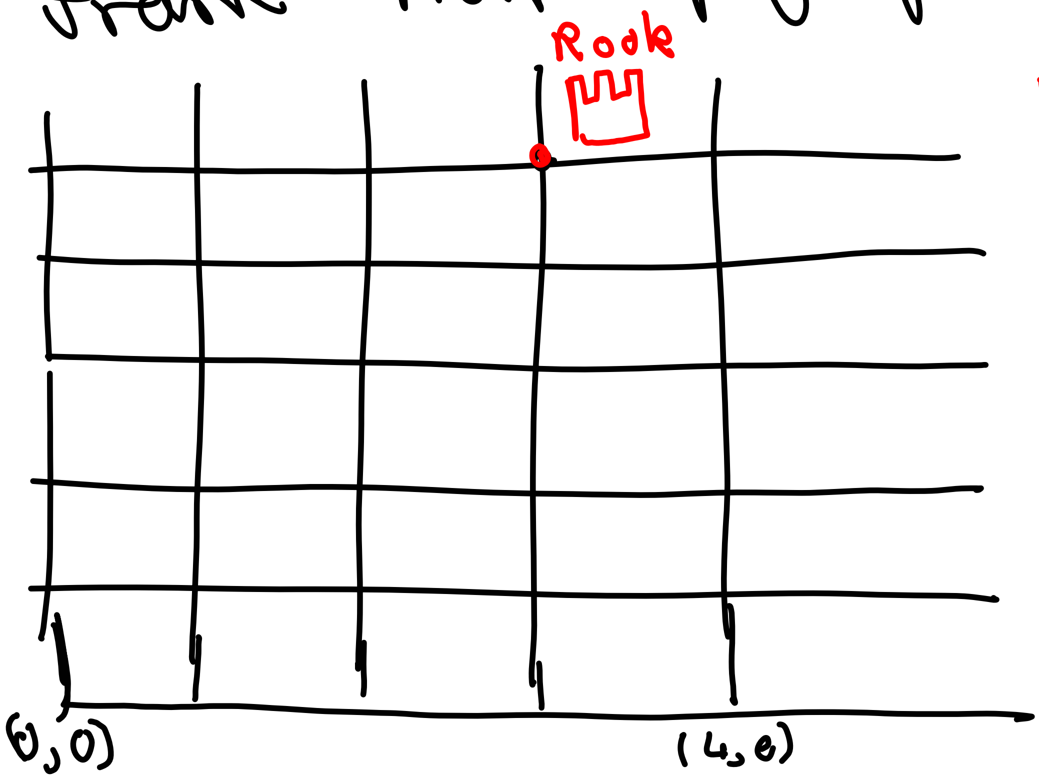
Losing positions are those divisible by 5.

Game 2



Harder Game: Put a Queen on board

Move: take non-zero # of chips from non-empty pile



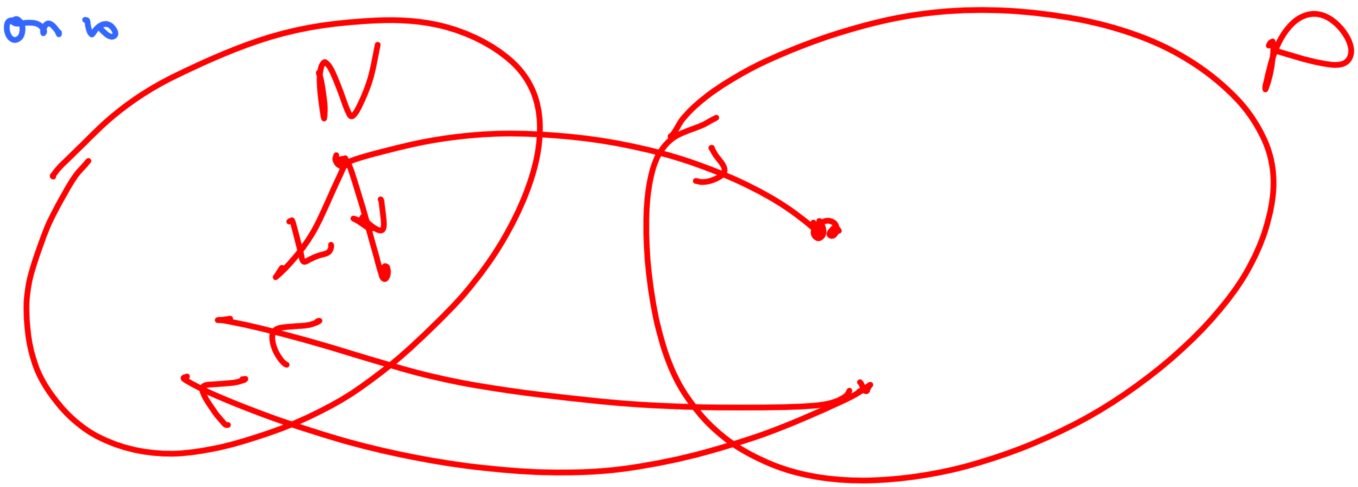
Diagonals are losing positions

In general positions can be

winning N → next player wins

losing P → previous player wins

If position is N, ∃ move to a P position



if position is P, all moves go to N

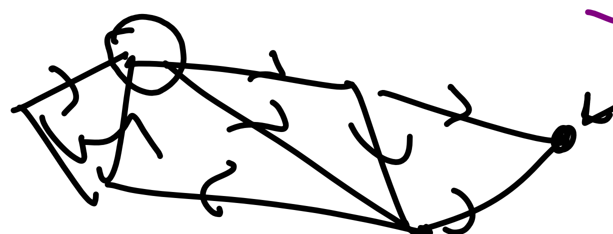
Abstraction of game

Represent each position by a node of a digraph $D = (X, A)$

X = {positions}

∃ move x to y

Game:



D

losing position

+ token ○

move ≡ move token along an edge.