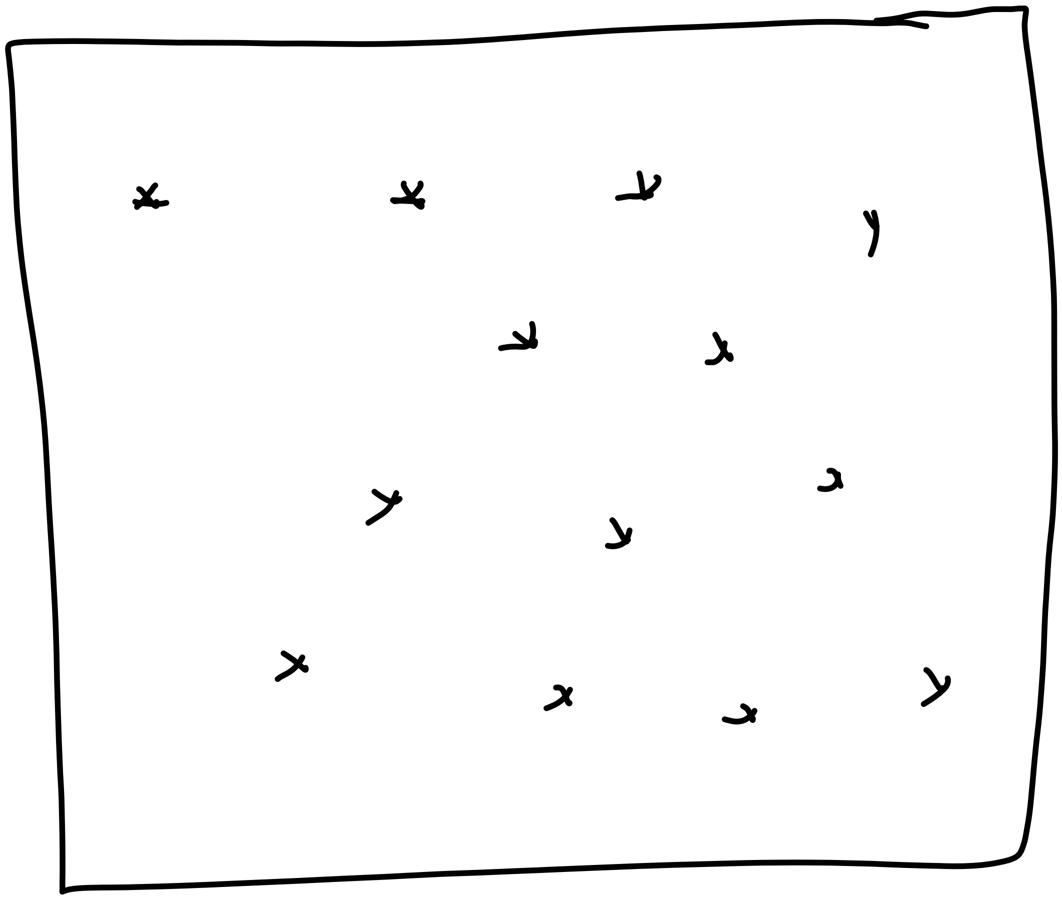
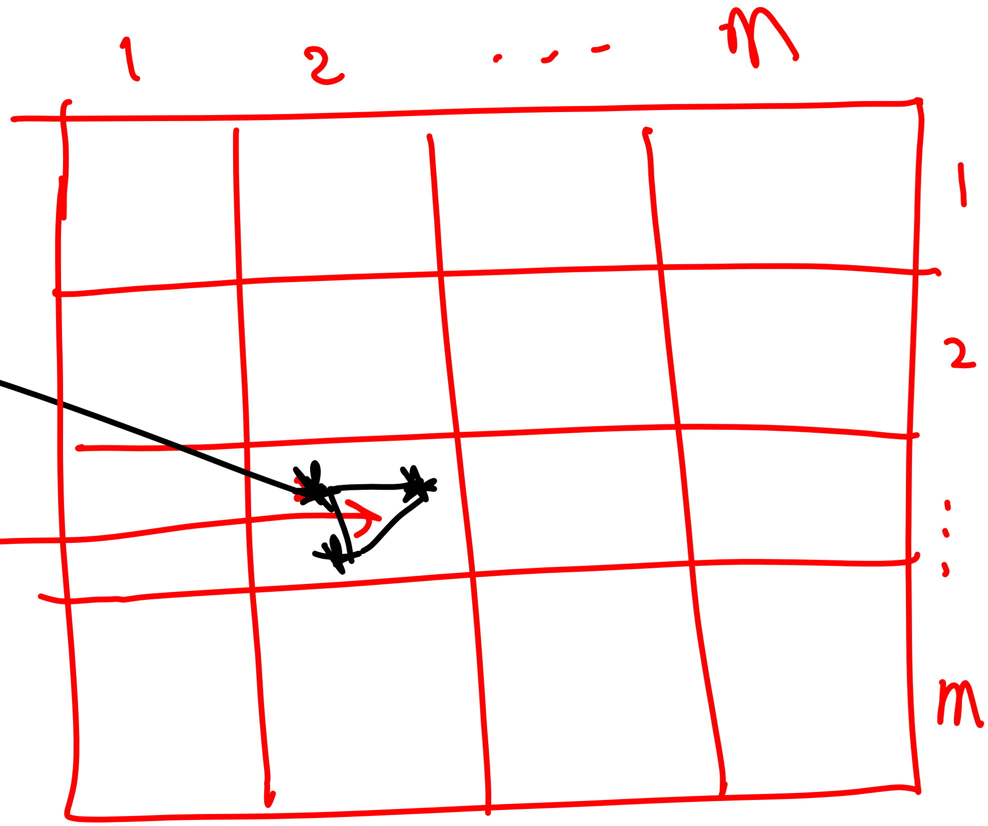
10\30\09

$[0,1]^2$



n points : $\exists$ triangle of area $\leq c\frac{1}{n}$

small area $\le \frac{1}{2m^2}$



$m \times m$ subsquare

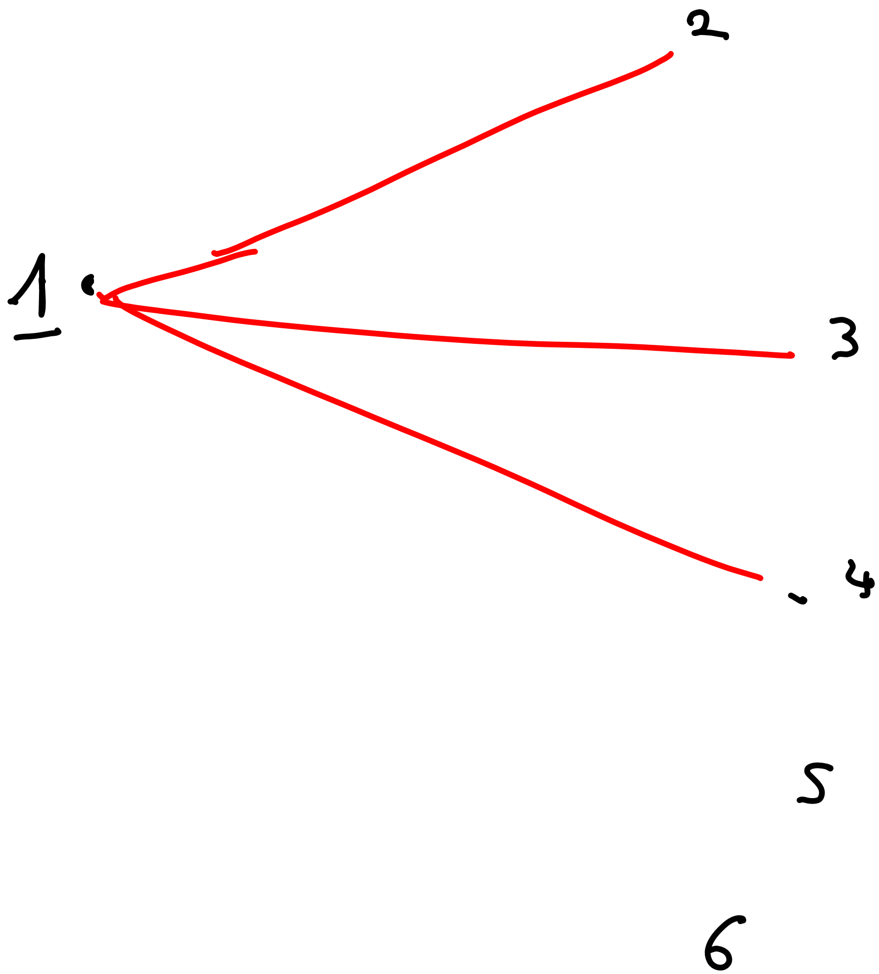
$m^2 < \frac{n}{2}$

$\exists$ hole with ≥ 3 pigeons

n pigeons $\equiv$ points

$< \frac{n}{2}$ holes $\equiv$ subsquare

Ramsey Theory (Frank Ramsey)



By PHP

$\exists$ 3 edges

of same

color

Either 23 & 24 & 34

are all blue

or

$\exists$ red $\triangle$

Ramsey's Theorem

For all positive integers k, l there exists $R(k,l)$ such if $N \geq R(k,l)$ and the edges of K_N are colored Red or Blue then either

(i) $\exists$ Red K_k

or

(ii) $\exists$ Blue K_l

$$R(1,k) = R(k,1) = 1$$

$$K_1 = \bullet$$

$$R(2,k) = R(k,2) = k$$

$k = 4$

Red K_2 or Blue K_4

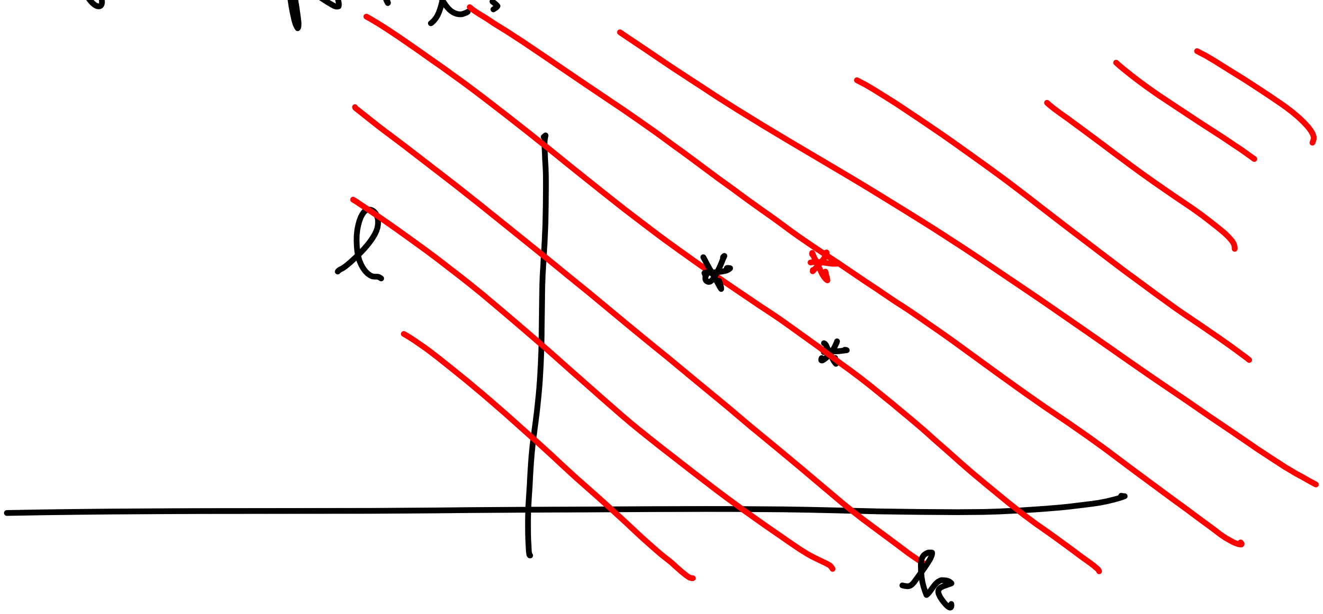
$$R(3,3) = 6$$

$$R(4,4) = 18$$

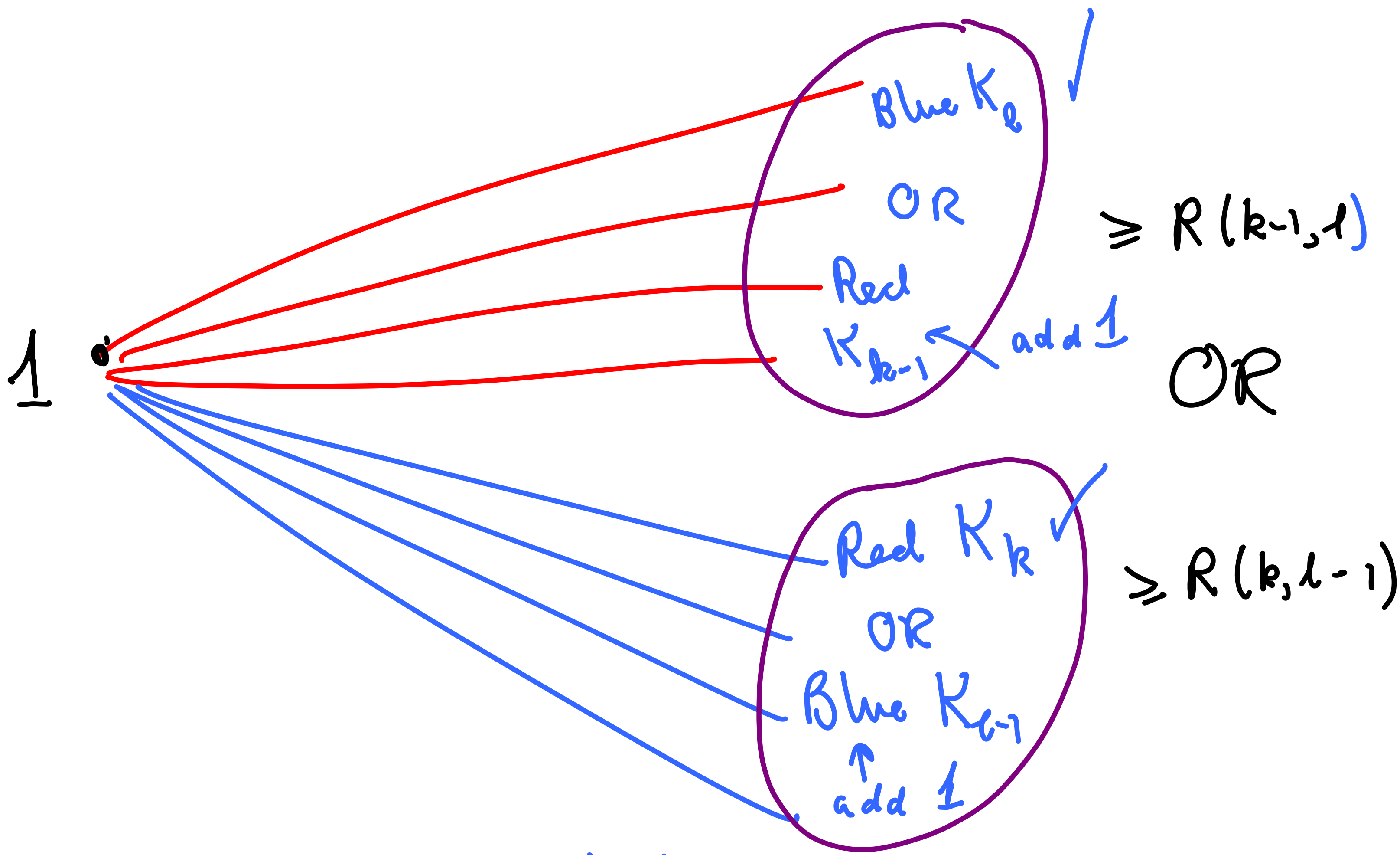
$$R(5,5) = ?$$

$$R(k,l) \leq R(k,l-1) + R(k-1,l)$$

This will prove that $R(k,l)$ exists for all k,l — use induction on $k+l$.



$$N = R(k, l-1) + R(k-1, l)$$



suppose $\leq R(k-1, l) - 1$ Red — only colored $\leq N-2$ edges

$\leq R(k, l-1) - 1$ Blue

$$R(k,l) \leq \binom{k+l-2}{k-1}$$

By induction on $k+l$

$$R(k,l) \leq R(k-1,l) + R(k,l-1)$$

$$\leq \binom{k+l-3}{k-2} + \binom{k+l-3}{k-1}$$

$$= \binom{k+l-2}{k-1}$$

$$R(k,k) \leq \binom{2k-2}{k-1}$$

$$< 4^k$$

$$R(k,k) > 2^{k/2}$$

Probabilistic Method:

If $N \leq 2^{k/2}$ then $\exists$ a 2-coloring of the edges of K_n without a Red K_k or a Blue K_k.

Proof, take a random 2-coloring.