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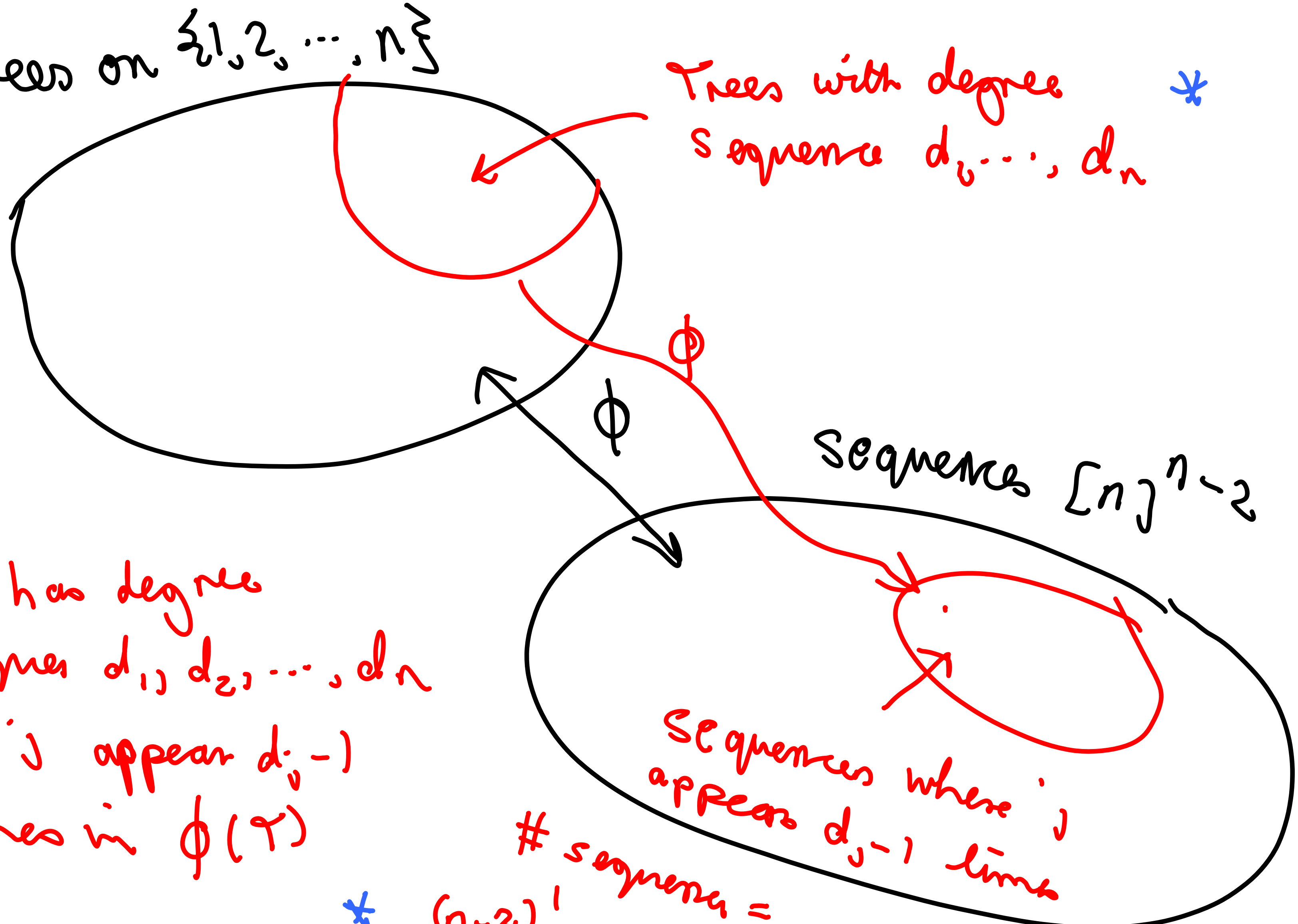


If T has degree sequence $d_1, d_2, \dots, d_n$ then i appear $d_i - 1$ times in $\phi(T)$

sequence =

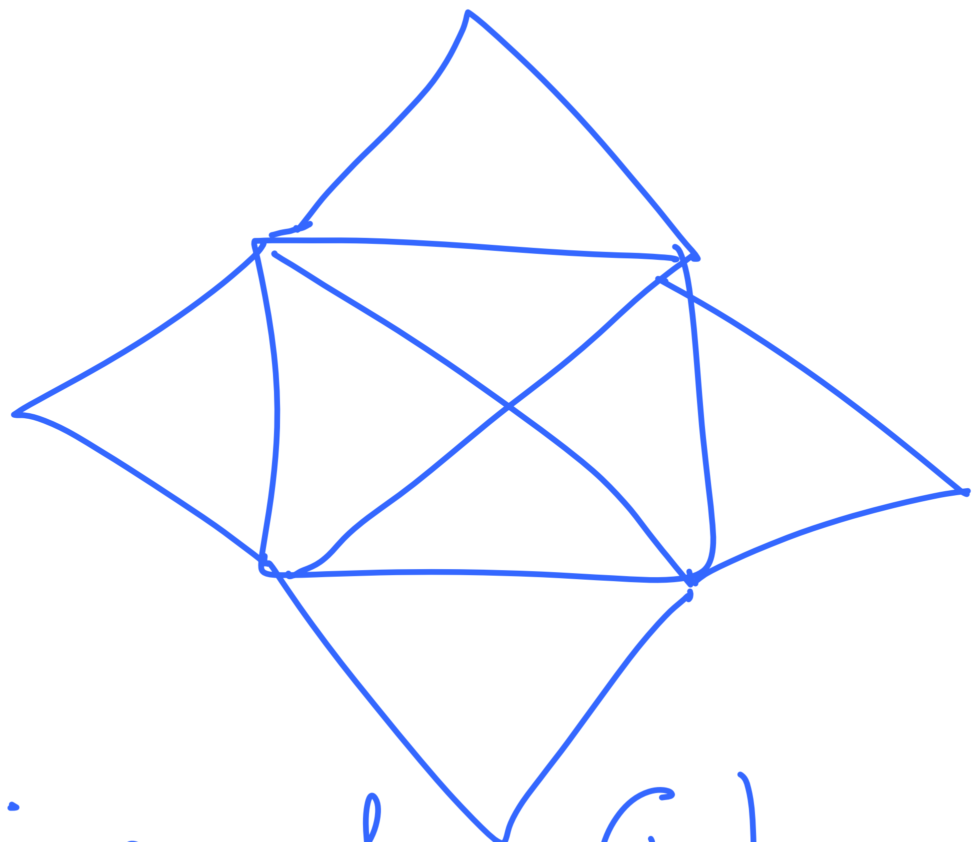
$$* \quad \frac{(n-2)!}{(d_1-1)! \cdots (d_n-1)!}$$

Euler Tours

$G = (V, E)$ is a connected graph.

An Euler tour H is a closed walk that visits each edge <u>exactly</u> once

G is Eulerian if it has an Euler tour.



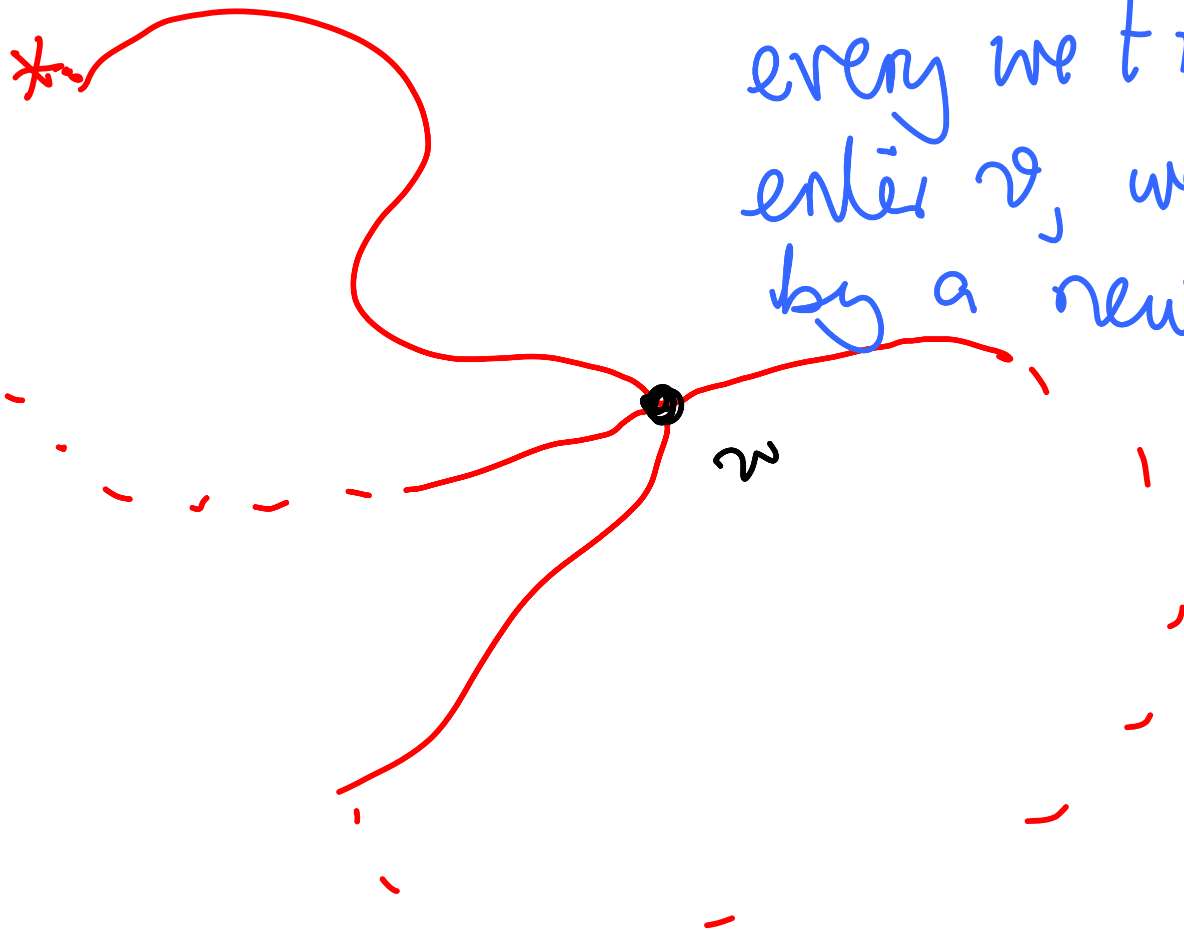
Is this graph Eulerian?

Thm

G is Eulerian iff every vertex has an **even** degree (and G is connected).

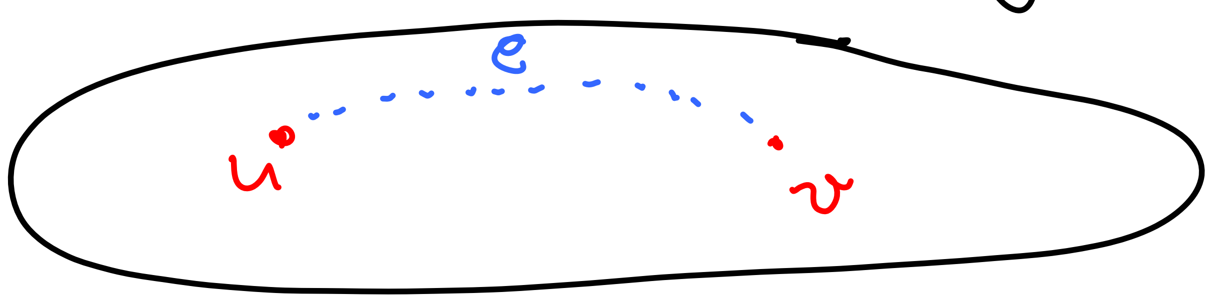
Eulerian $\Longrightarrow$ even degrees.

every we time we enter v, we must leave by a new edge.



Euler path is a walk that goes through every edge

G (connected) has an Euler path iff G has at most 2 odd degree vertices



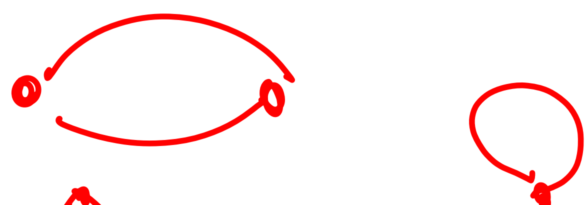
$G + e$

Even degrees (Connected) $\Rightarrow$ Euler Tour.

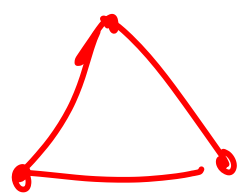
Proof

By induction on $|E|$.

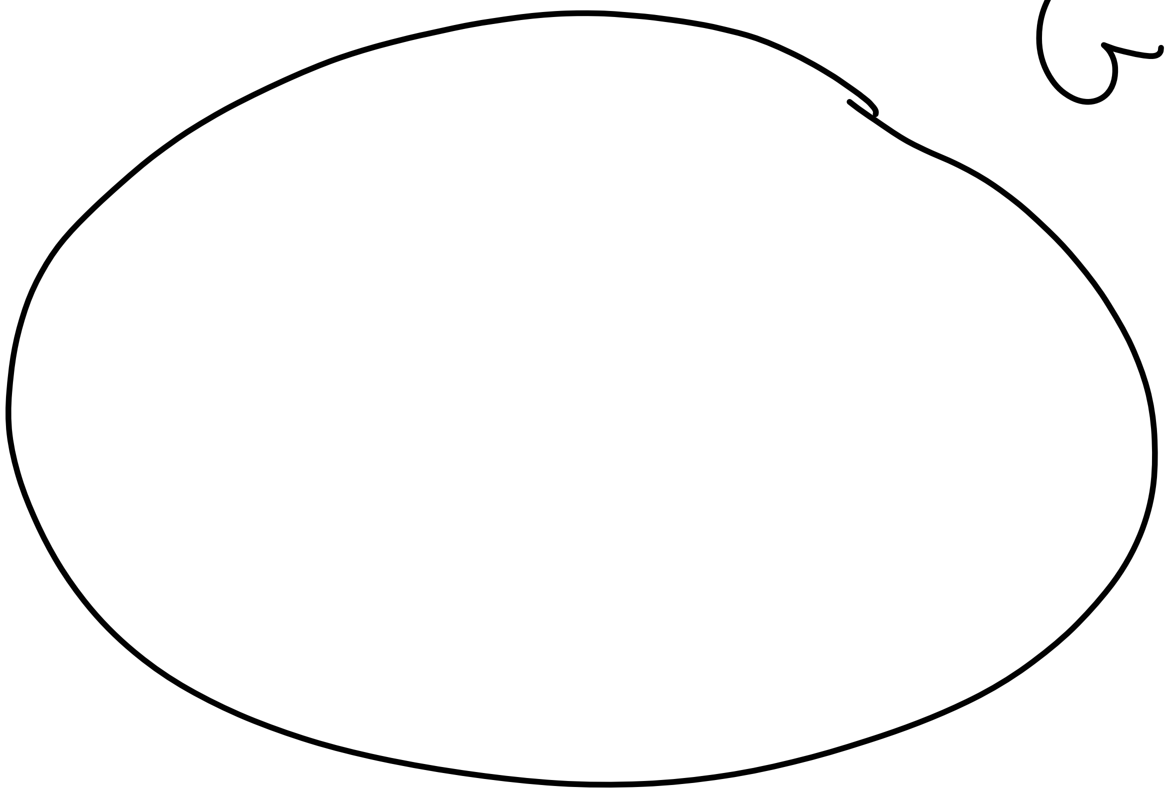
Base Case:



or



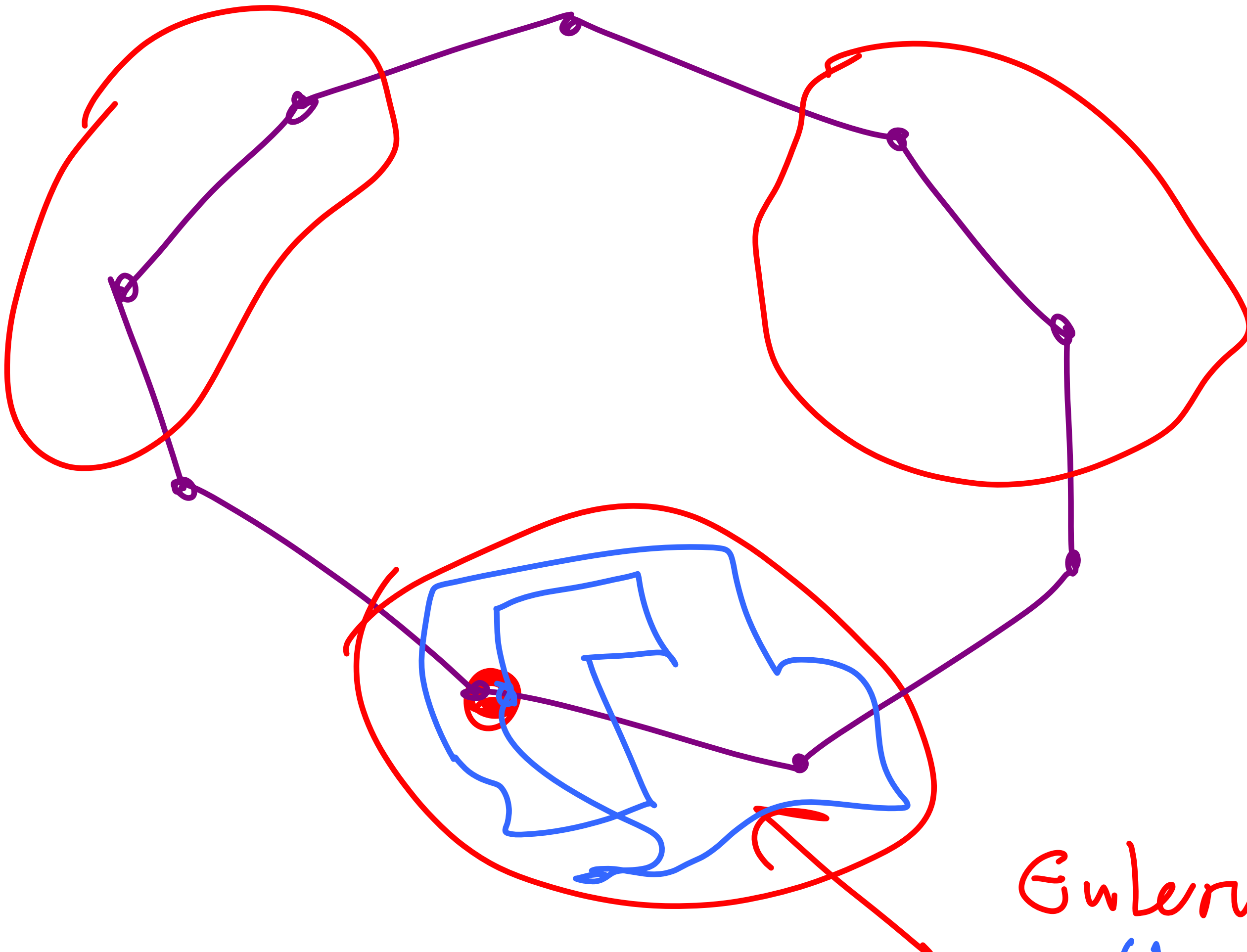
G is eulerian



G has a cycle C. –

if G was acyclic then G would be a tree and have a vertex of degree 1

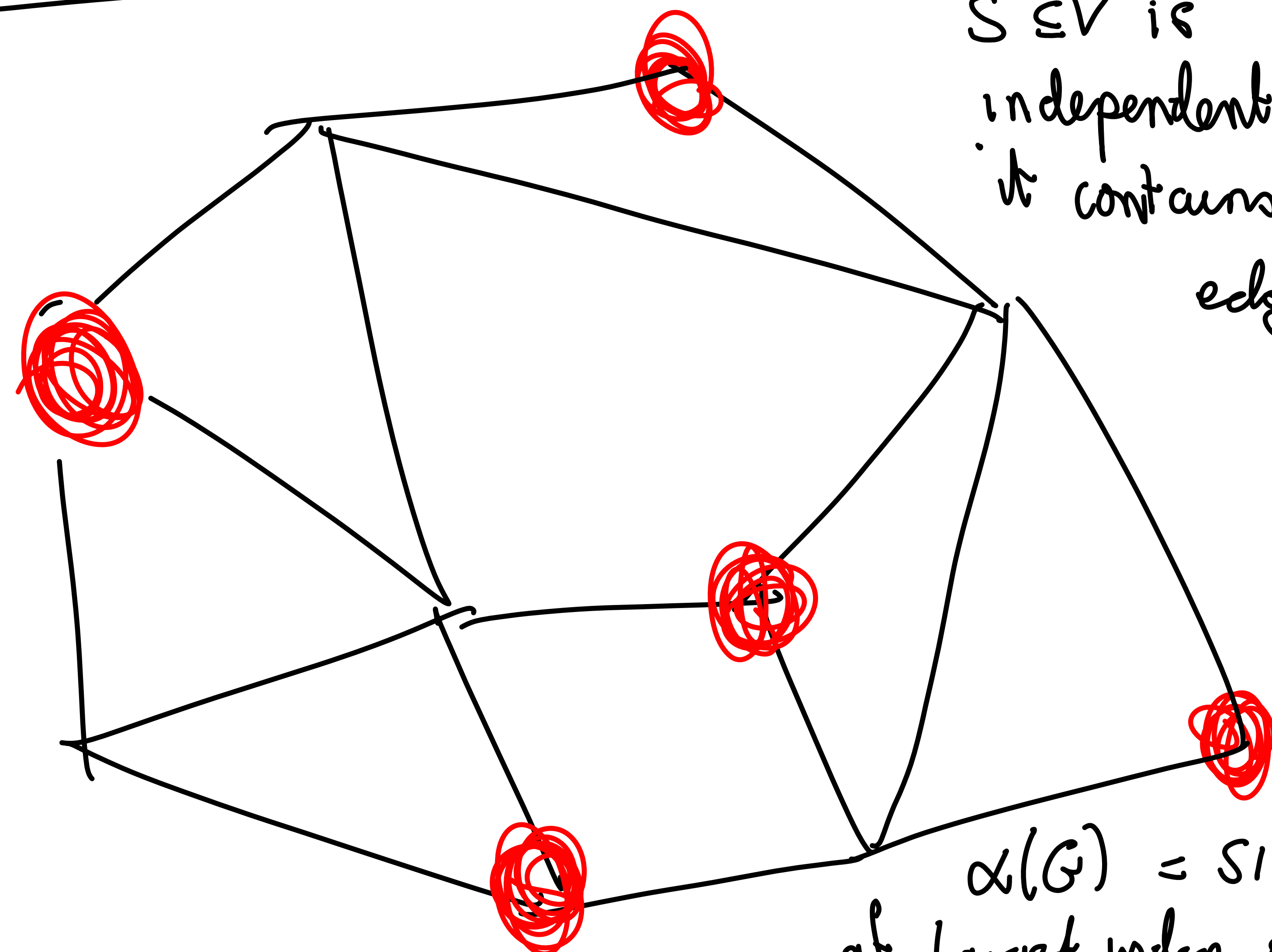
$G - E(C)$ only has even degrees.



Eulerian

so (by induction) has an euler tour

Independent Sets and Cliques



$S \subseteq V$ is independent if it contains no edge.

$\alpha(G)$ = size of largest indep set

Suppose $|V| = n$

Max. degree $= \Delta$

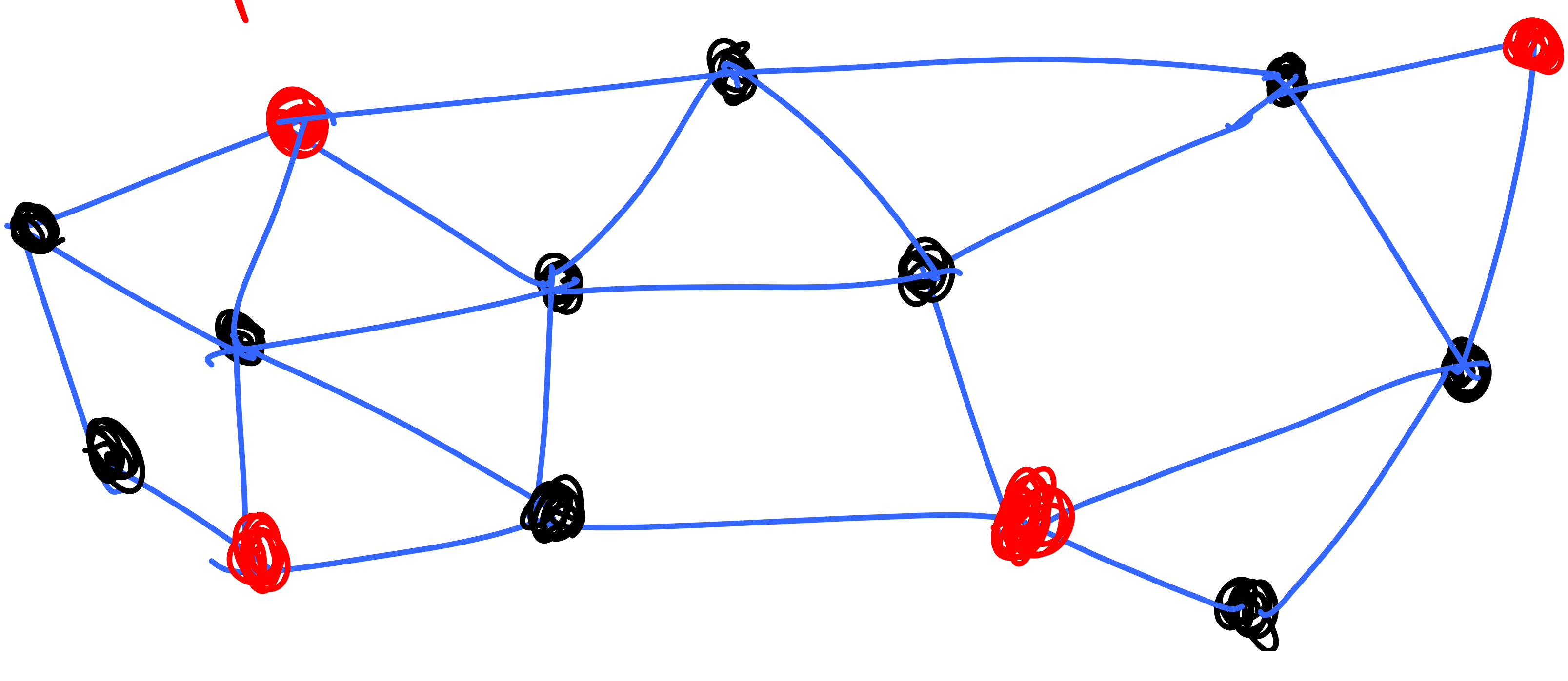
$$\alpha(G) \geq \frac{n}{\Delta + 1}$$

Greedy Algorithm:

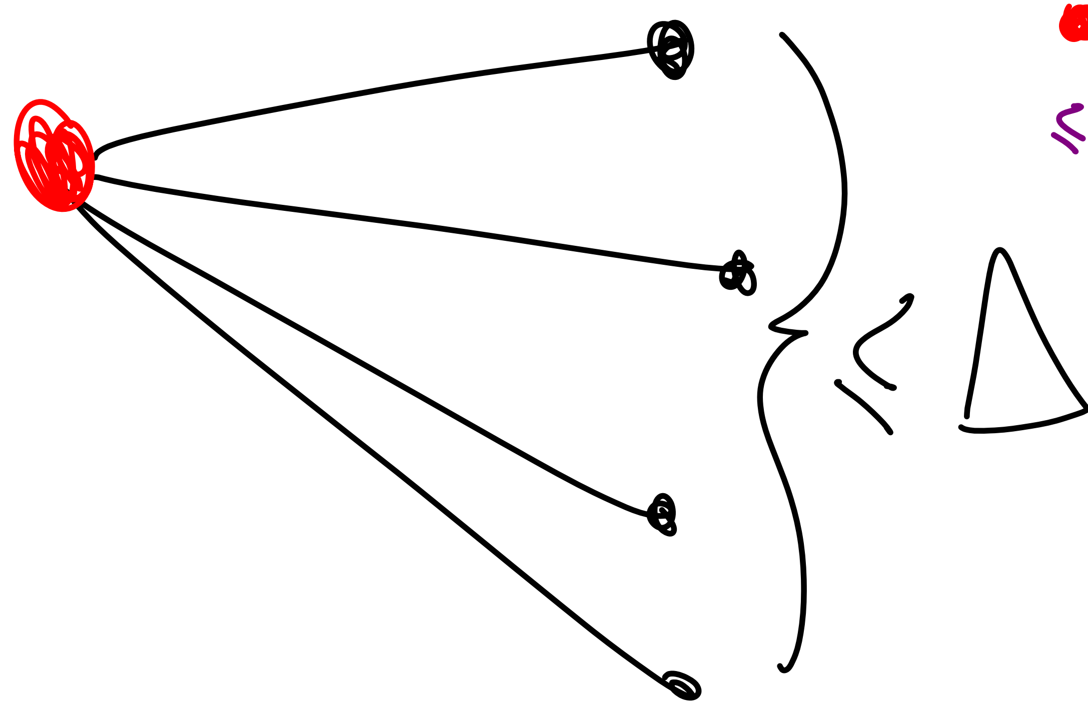
Choose vertex v, add it to S

Delete nbrs of v

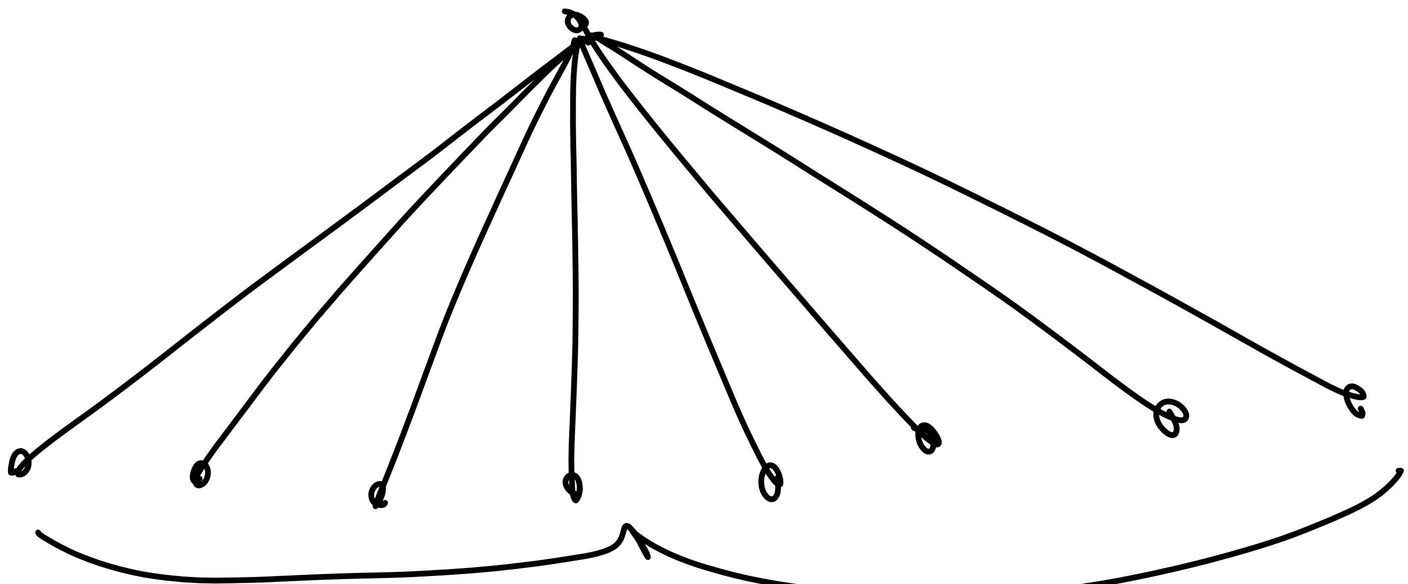
Repeat



Greedy produces a a independent set of size $\geq \frac{n}{\Delta+1}$



Each choice of removes $\leq \Delta+1$ vrtcs.



$\Delta = n-1$

$\alpha(G) \geq 1$

$$\alpha(G) \geq \frac{n}{\cancel{\Delta}+1}$$

average degree

Turan's Theorem