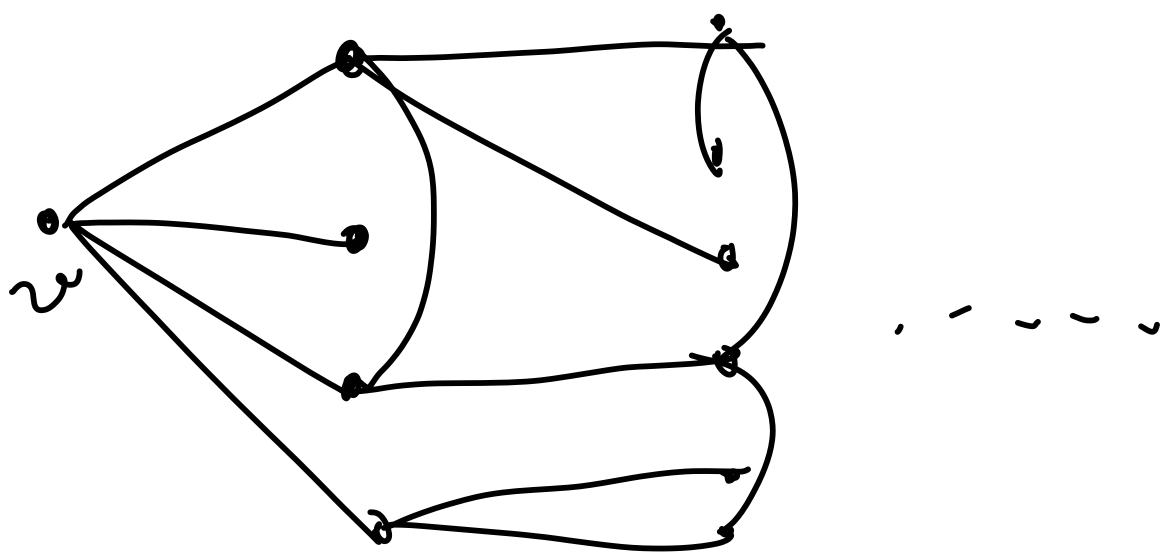
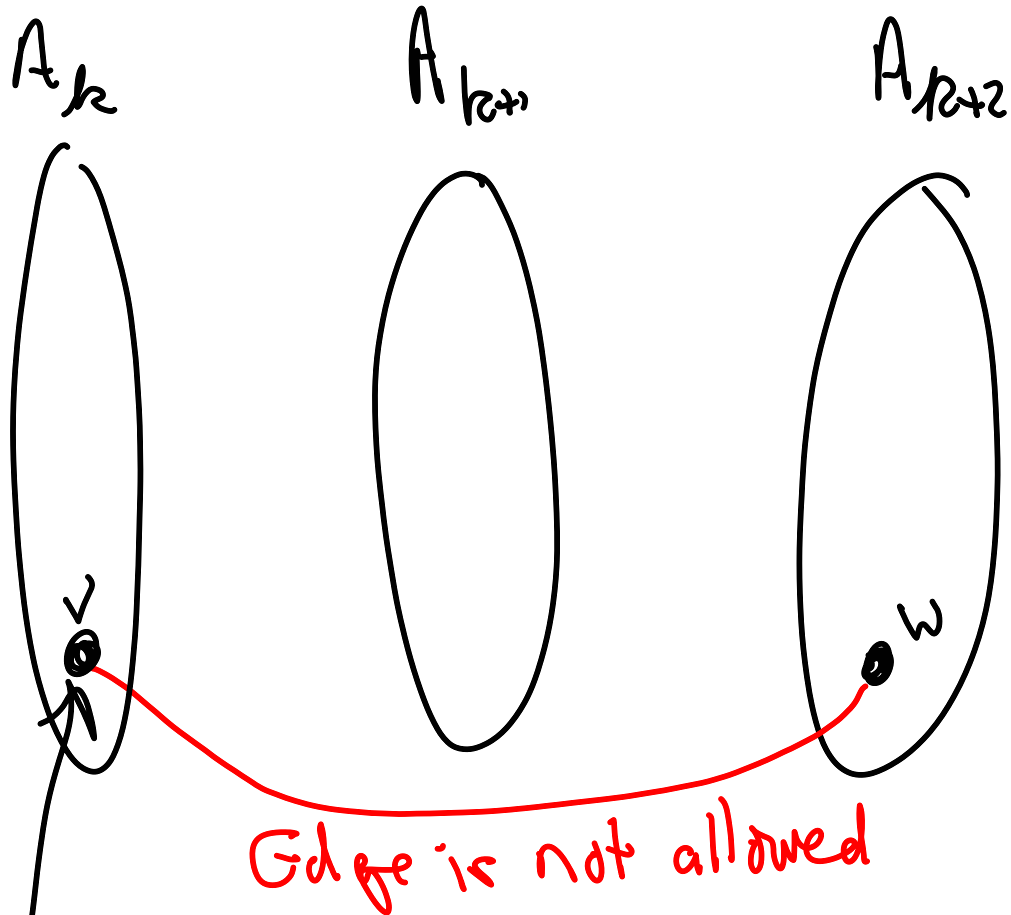
10/19/09

Breadth First Search BFS



A_0 A_1 A_2

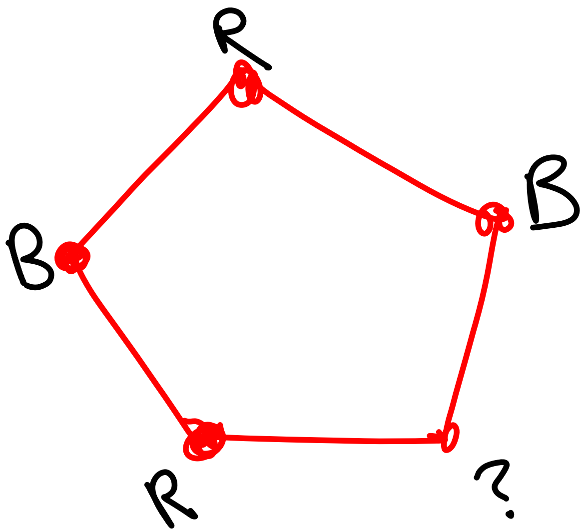
"explore" component containing v.



vertices at distance k from v.

shortest

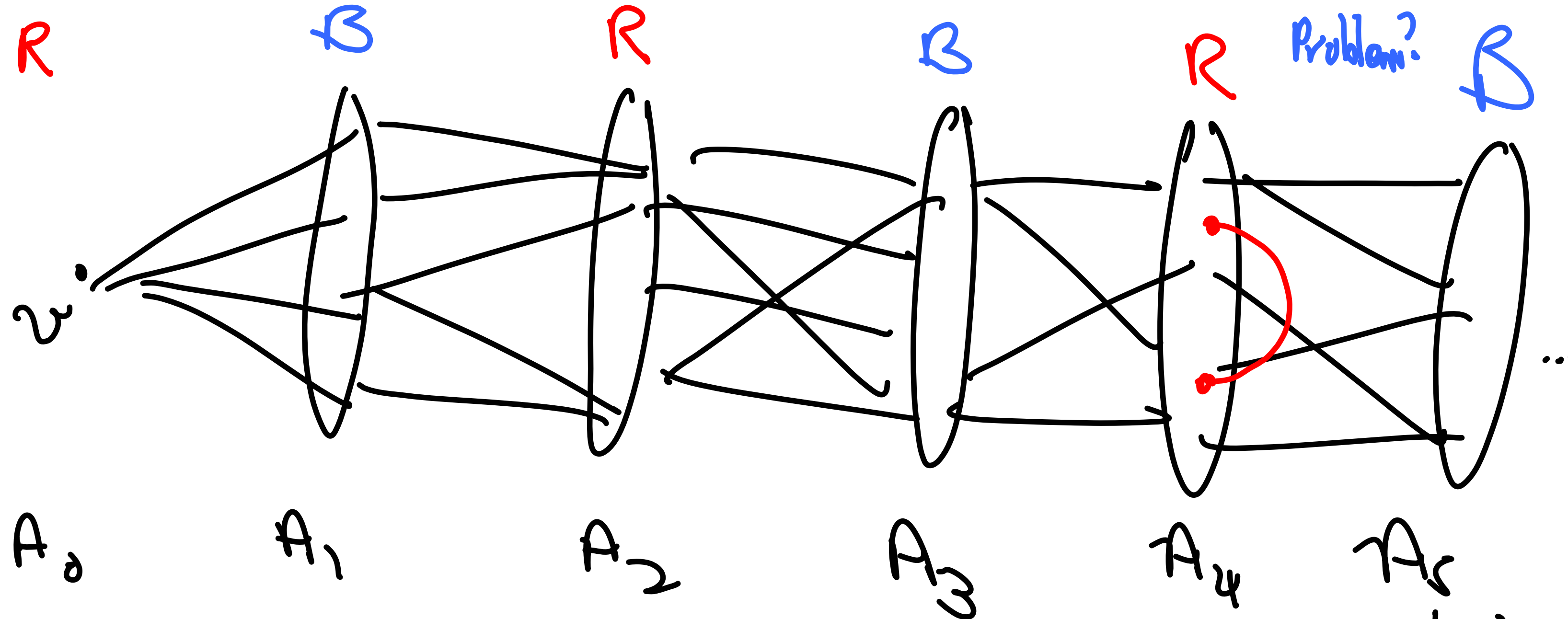
When is a graph bipartite?



← non bipartite

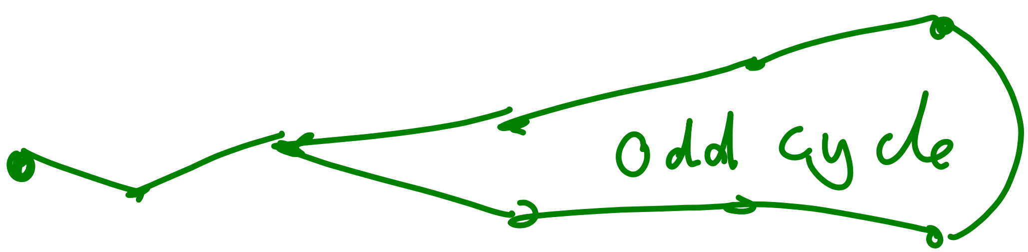
An odd cycle is non-bipartite.

So graphs which contain odd cycles are non-bipartite



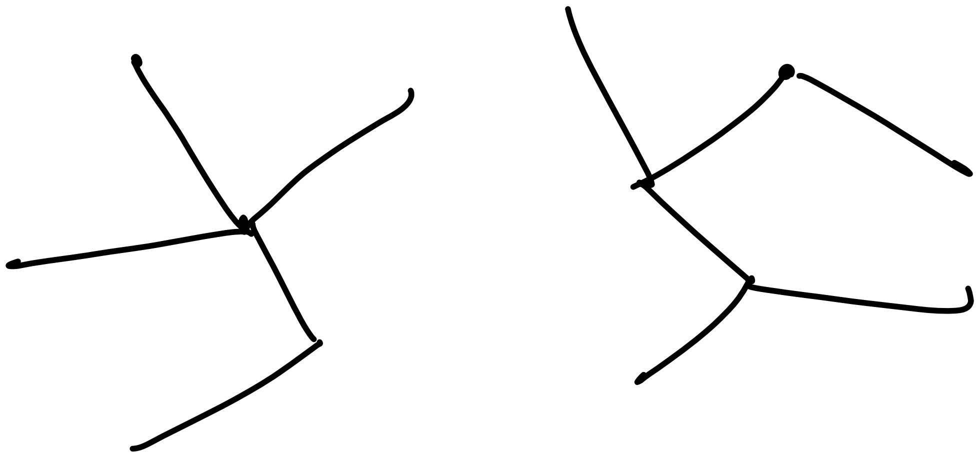
CLAIM: no odd cycles $\Rightarrow$ bipartite.

Why no



Assume no odd cycles, $\Rightarrow$ no

Trees

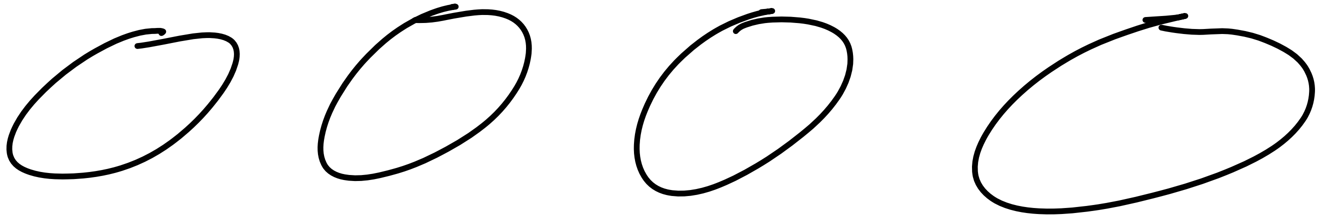


A tree is a graph which is

(i) Connected
(ii) No cycles

Lemma

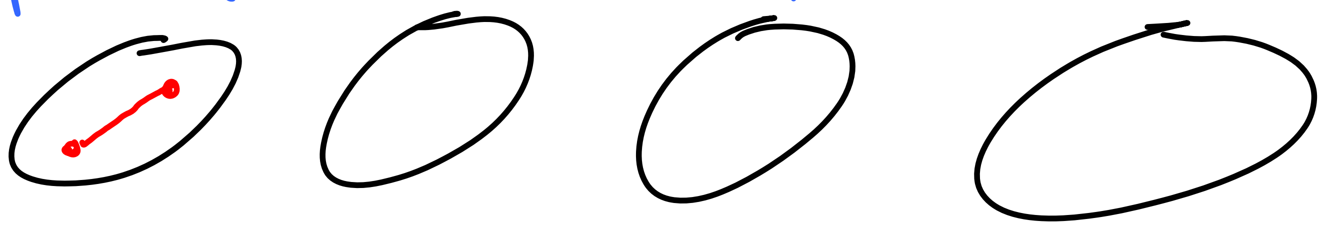
component

G = 

Add an edge

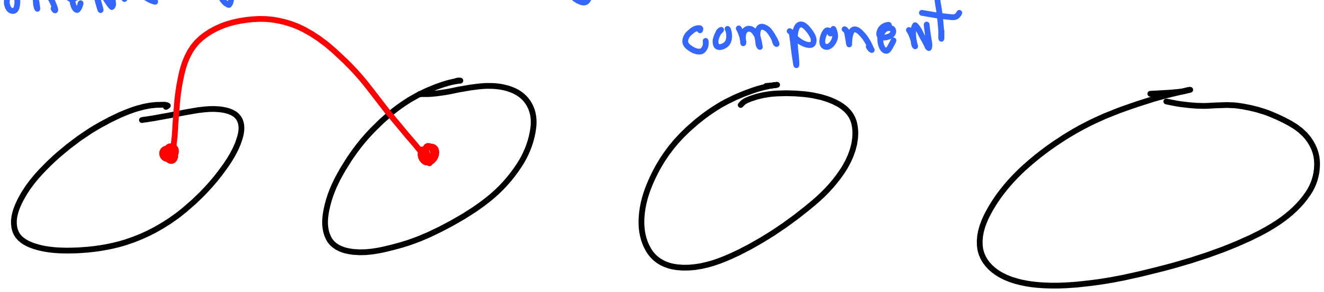
#comps. stays same.

component

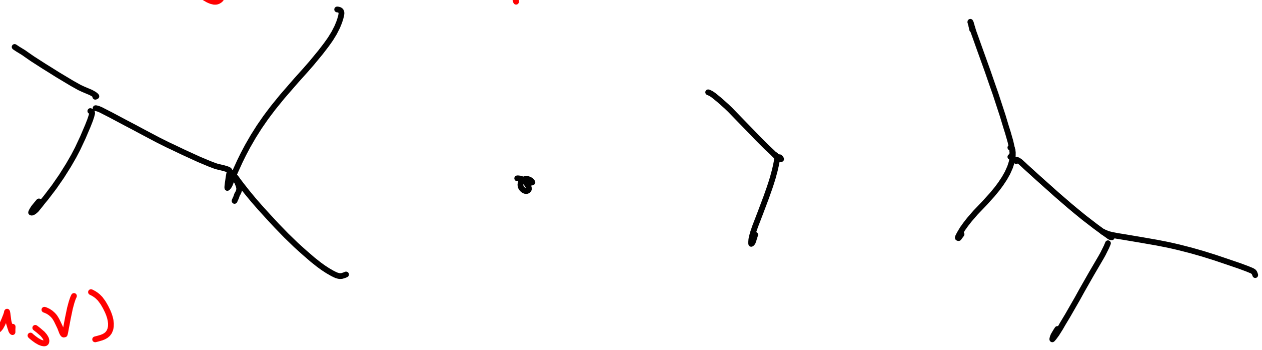
G = 

#components goes down by 1.

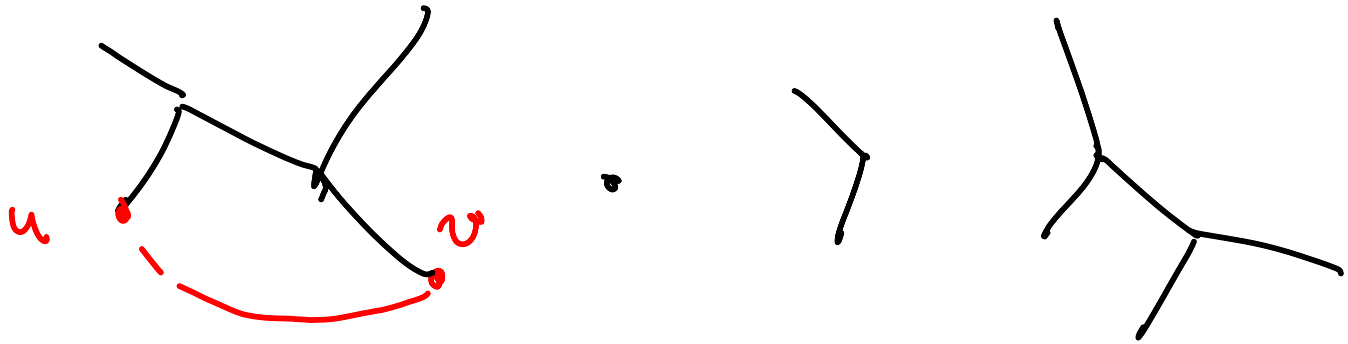
component

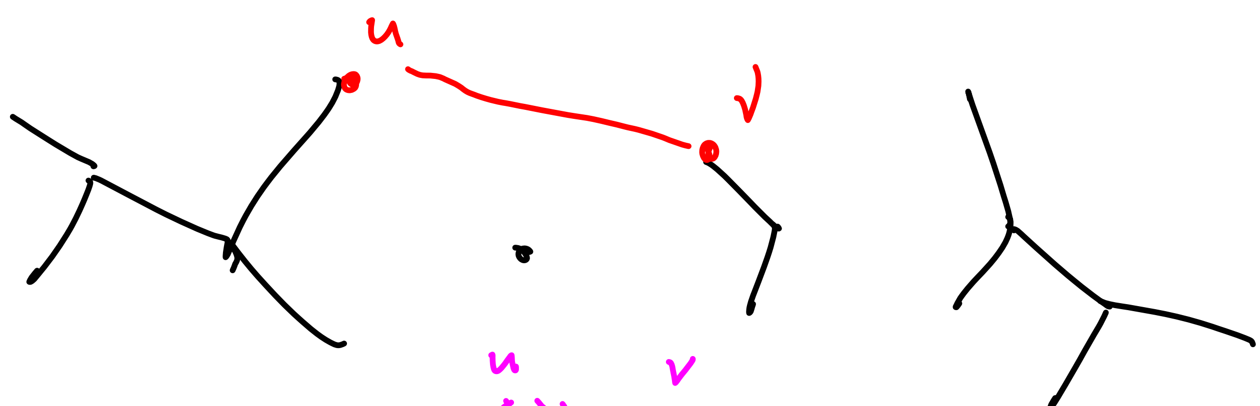
G = 

G : has k acyclic components



Add e = (u,v)





— not possible

G has no cycles &

k componenents.

If I add (u,v) then either

A1: u,v in same component; make cycle.
k components.

OR

A2: u,v in different components; no cycle
one less component

Suppose G has k components

$$\Downarrow$$

G has $n-k$ edges.

$G = (V, E)$ $\quad E = \{e_1, e_2, \dots, e_\ell\}$

$|V| = n$

G_0 has n comps.

$\vdots$

G_i has $n-i$ comps

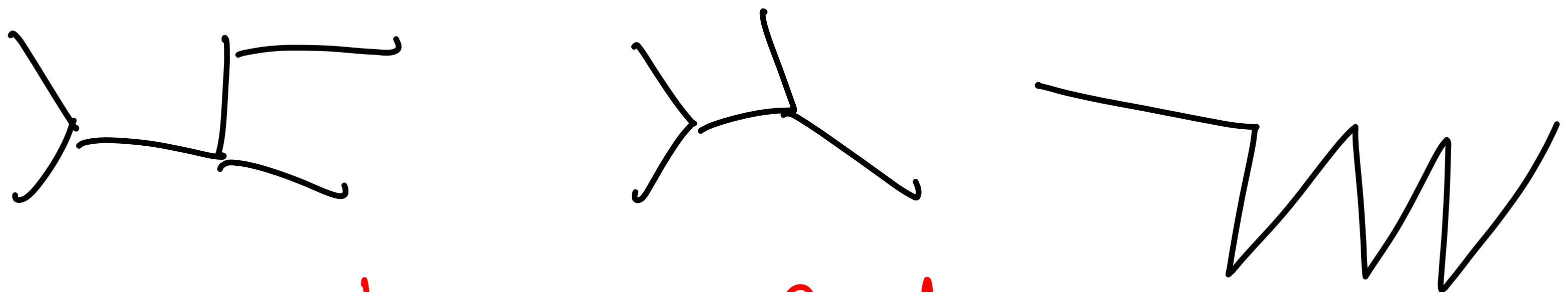
G_ℓ has $n-\ell = k$ comps.

$G_i = (V, \{e_1, e_2, \dots, e_i\})$

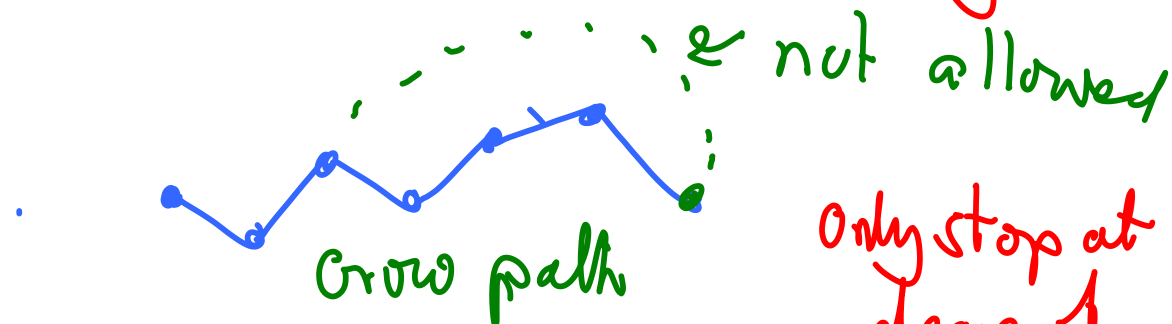
G_i has one less component than G_{i-1}

G_{i-1} was acyclic, adding e_i was A2

A tree with n vertices has $n-1$ edges.



≥ 2 vertices of degree 1.



Only stop at vertex of degree 1.

Theorem

Suppose $|V| = n$, $|E| = n-1$. Then following are equivalent.

(a) Connected

(b) Acyclic

(c) Tree

$E = \{e_1, e_2, \ldots, e_{n-1}\}$

$G_i = (V, \{e_1, \ldots, e_i\})$