10\14\09

$$M = \begin{array}{c} \\ v_1 \\ v_2 \\ v \\ \vdots \\ v_m \end{array} \begin{array}{c} \begin{array}{ccc} & e & \end{array} \\ \begin{bmatrix} & & \\ & & \\ 1 & 1 & 1 \\ & & \\ & 1 & \end{bmatrix} \end{array} \; d_G(v)$$

$$\sum_{v \in V} d_G(v) = 2|E|$$

\# of 1's in M

sum by rows

sum by columns

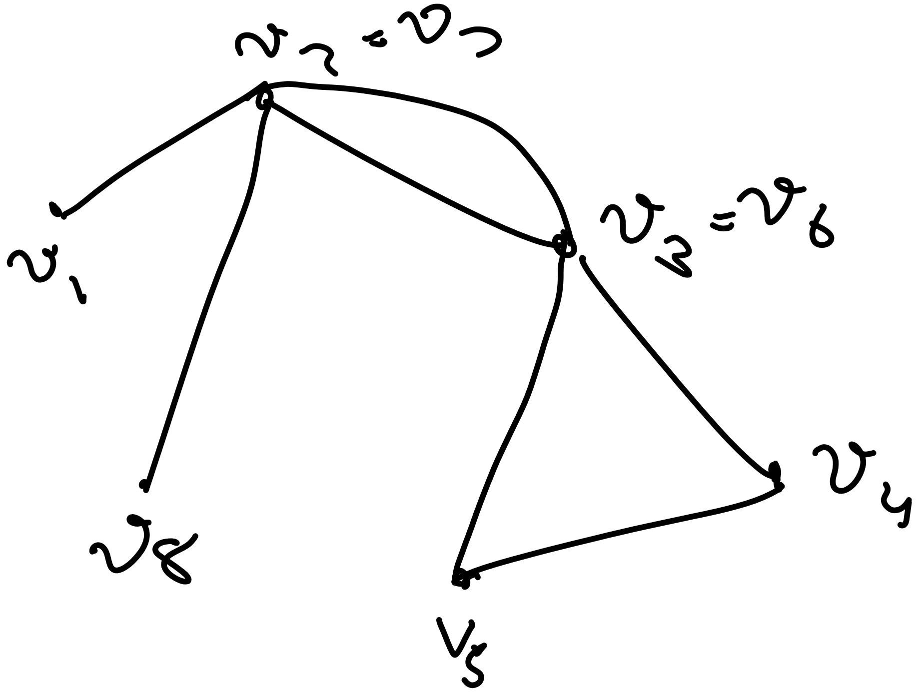
"Double Counting"

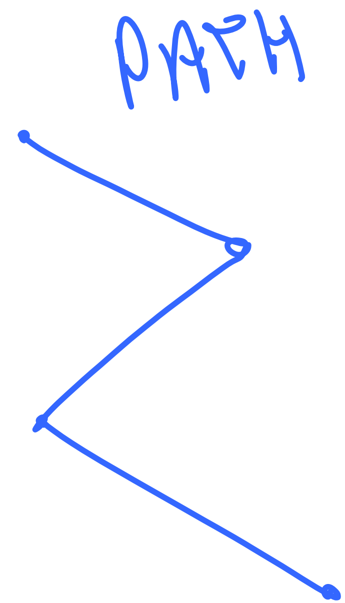
Corollary

of vertices of odd degree is even.

$$\sum_{v \in ODD} d(v) = 2|E| - \sum_{v \in EVEN} d(v)$$

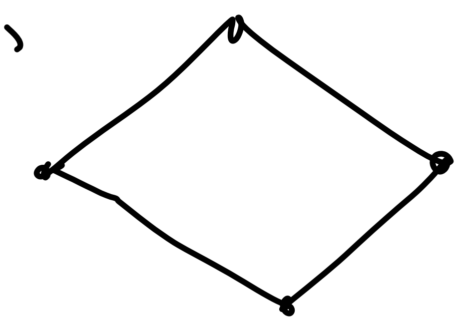
Paths & Walks





A path is a walk that goes through a vertex at most once.

A walk is closed if it begins and ends at same vertex,

A cycle : 

No Repetition of vertices

Thm

A = adjacency matrix of G.

$M_k = A^k$

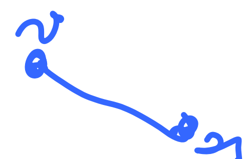
$M_k(v, w)$ = # of walks from v to w of length k.

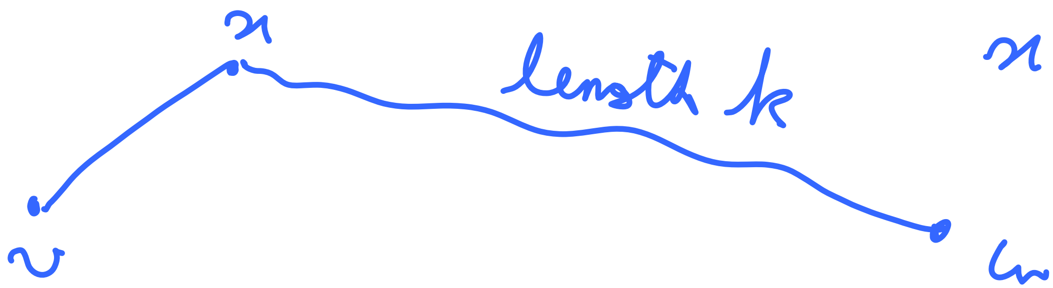
Proof Induction on k.

True for $k = 1$. (Base Case).

$M_{k+1} = A M_k$

$$M_{k+1}(v,w) = \sum_{x} \underbrace{A(v,x)}_{\text{\# edge}} \; \underbrace{M_k(x,w)}_{\text{\# walks of length } k \text{ from } x \text{ to } w}$$

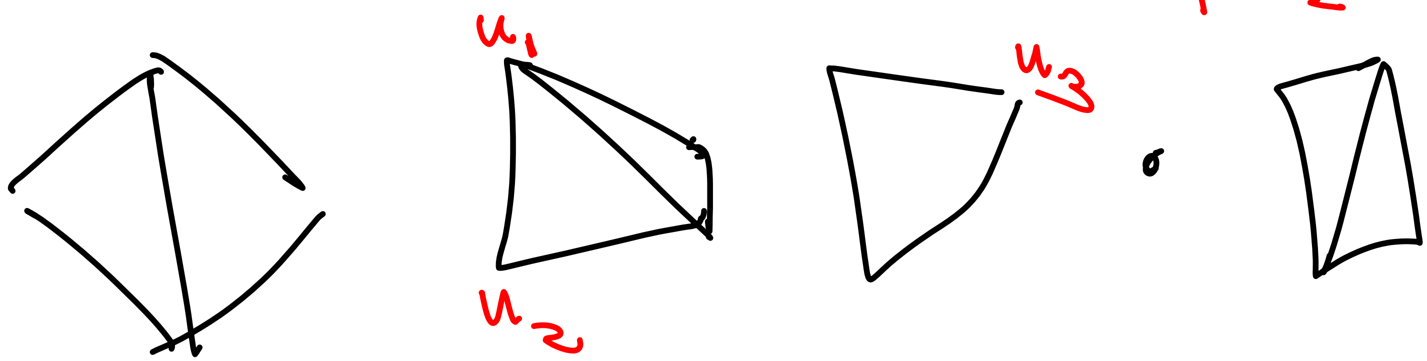




Cannot count number of paths of length k in this manner.

Connected Components

$u_1 \sim u_2$ $\quad u_1 \not\sim u_3$



One graph: 5 pieces

components

Say $u \sim v$ if $\exists$ walk from u to v

$\sim$ is an equivalence relation.

Reflexive $\quad u \sim u$

Symmetric $\quad u \sim v \Rightarrow v \sim u$

Transitive $\quad u \sim v$ & $v \sim w \Rightarrow u \sim w$

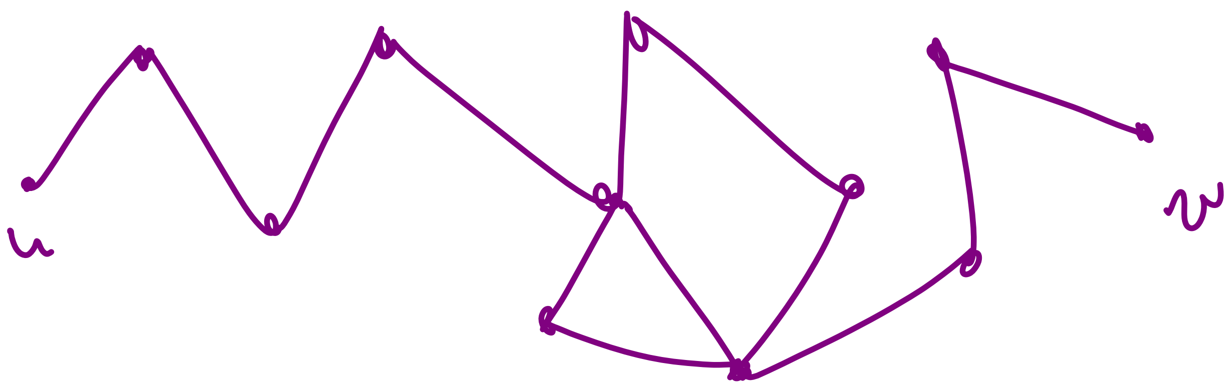
Equivalence classes: Connected Components

A graph is connected if there is one component i.e.

$u \leadsto v \quad \forall u,v$

Simple Observation.

If $\exists$ a walk from u to v then there is a path from u to v.



Take a shortest walk u to v.

Remove to get a shorter walk.