

21-301 Combinatorics

Homework 9

Due: Wednesday, November 19

1. Consider the following game: There is a pile of n chips. A move consists of removing any *proper* factor of n chips from the pile. (For the purposes of this question, a proper factor of n , is any factor, including 1, that is strictly less than n). The player to leave a pile with one chip wins. Determine the N and P positions and a winning strategy from an N position.
2. Consider the following game: There is a single pile of n chips. A move consists of removing (i) any *even* number of chips provided it is not the whole pile, or (ii) the whole pile, but only if it has $2 \pmod{3}$ chips. The terminal positions are zero and one. Determine the Sprague-Grundy numbers of each pile size.
(Compute the first 15 numbers and see if you can see a pattern.)
3. In a take-away game, the set S of the possible numbers of chips to remove is finite. Show that the Sprague-Grundy numbers satisfy $g(n) \leq |S|$ where n is the number of chips remaining.