

10/06/08

m balls (distinguishable)

n boxes [\equiv colors]

$Z = \#$ non-empty boxes

$$= Z_1 + Z_2 + \dots + Z_n$$

$$Z_i = 1 - \text{non-empty}$$

$$E(Z) = n E(Z_1)$$

$$= n \Pr(Z_1 > 0)$$

$\Pr(\text{Box 1 is non-empty})$

$$= 1 - \Pr(\text{Box 1 is empty})$$

$$= 1 - \frac{(n-1)^n}{n^n}$$

Distinguishable \checkmark Indistinguishable

3 balls

2 colors

DISYINGU... INDICR..

R R R \leftarrow 3R 0B

R R B \leftarrow 2R 1B

R B R \leftarrow 1R 2B

R B B

R R R

R R B

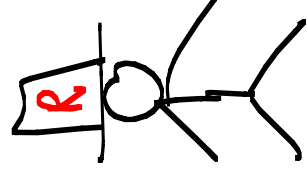
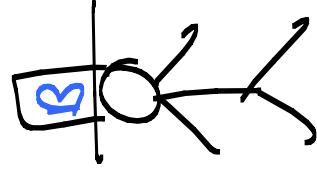
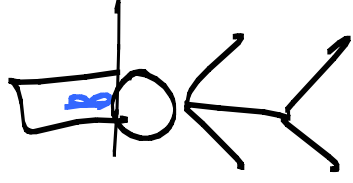
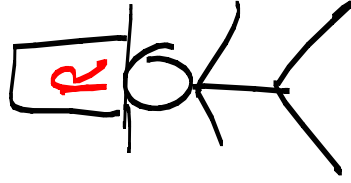
B B R

B B B

3 B



n people with hats on
their heads.



Hats
have
been
randomly
colored
Red / Blue

Problem:

Each Person can say

(I) My hat is Red

(II) My hat is Blue

(III) I don't know

If someone guesses their hat color

and no one guesses wrong color

BIG WIN

BIG LOSS if guess wrong color.

Claim: \exists strategy s.t.

$$Pr(\text{winning}) = 1 - o(1)$$

$Q_n = \{0, 1\}^n$ — stands for
R B hat colors

A coloring is an element of Q_n

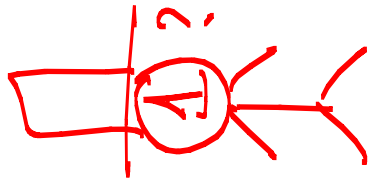


People enter room "sharing"

or fixed L, W - L is a const.

Every one "assumes" hat coloring is

in W . [$|W| \gg |L|$]



Is there a

unique value

hat color

that puts me

in W . . .

o

o

o

o

o

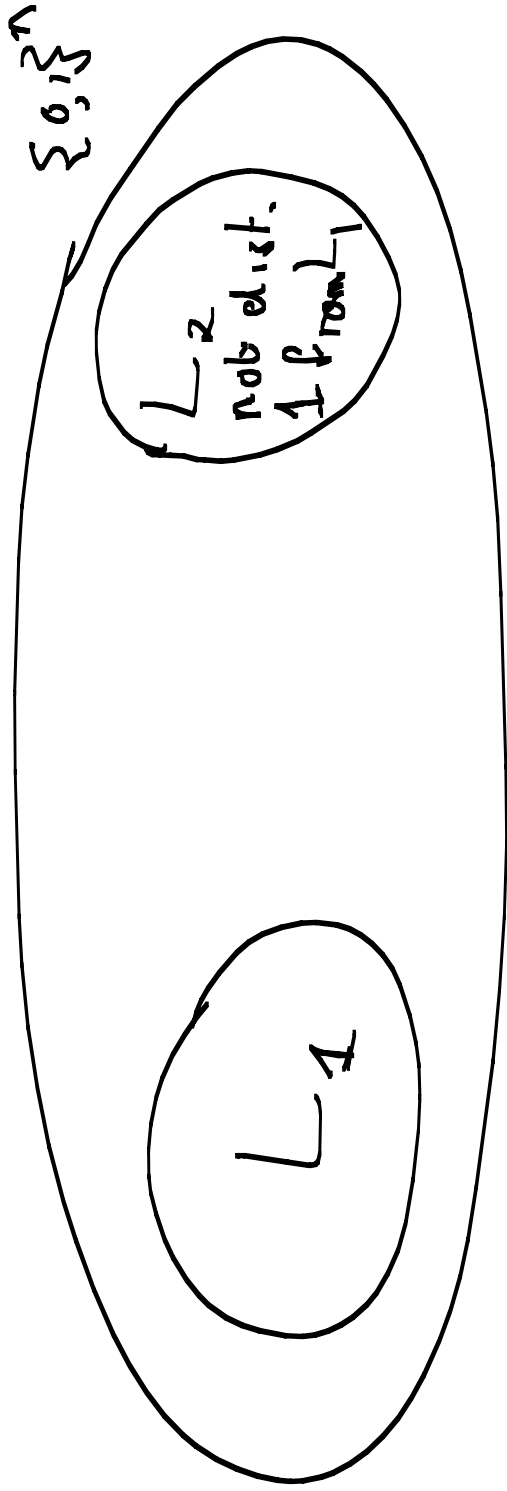
o

o

Is there a small L ?

$$\text{Let } p = \frac{\ln n}{n}$$

Choose L_1 randomly by placing
dc into L_1 with probability p .

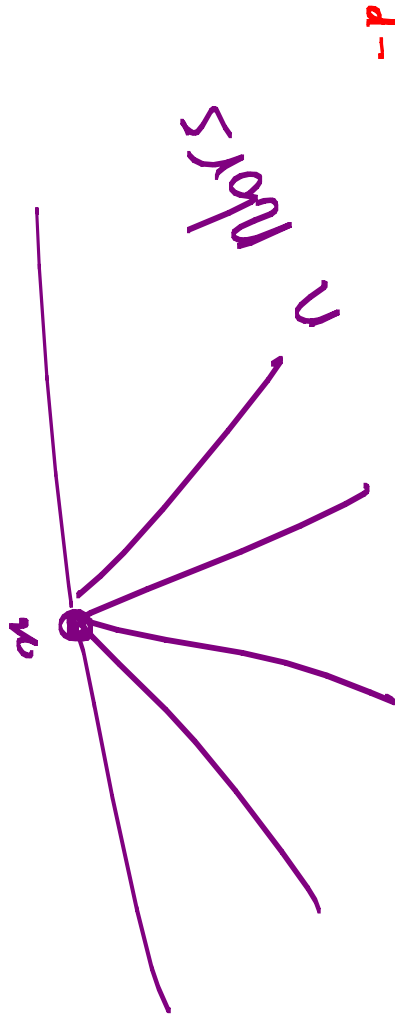


$$E(L_1, vL_2)$$

$$= E(L_1, 1) + E(L_2)$$

$$\underbrace{\hspace{10em}}_{2^n p} \quad \underbrace{\hspace{10em}}_{2^n (1-p)^{n+1}}$$

$$2^n p + 2^n (1-p)^{n+1}$$



$$\leq 2^n p + 2^n e^{-np} \leq 2^n \cdot \frac{2 \log n}{n}$$

$$1-p \leq e^{-p}$$

$$E(121) \leq 2^n \times \frac{2 \log n}{n}$$

\Rightarrow \mathcal{C} cover of size

$$\leq 2^n \times \frac{2 \log n}{n}$$

otherwise $E(121) > 2^n \times \frac{2 \log n}{n}$

Union Distinct Families

\mathcal{F} is a collection of subsets

$$\mathcal{F} \subseteq [n]$$

\mathcal{F} is union distinct if

$$A \cup B \neq C \cup D$$

for distinct A, B, C, D

We obtain a large family by

choosing p random sets.