10\27\08

Pigeon Hole Principle

$x_1, x_2, \ldots, x_m$ are non-negative integers then

$$\max \geq \lceil \text{average} \rceil$$

If $x_1 + \cdots + x_m \geq m+1$

$$\Rightarrow \exists i : x_i \geq 2$$

$A \subseteq [2n]$

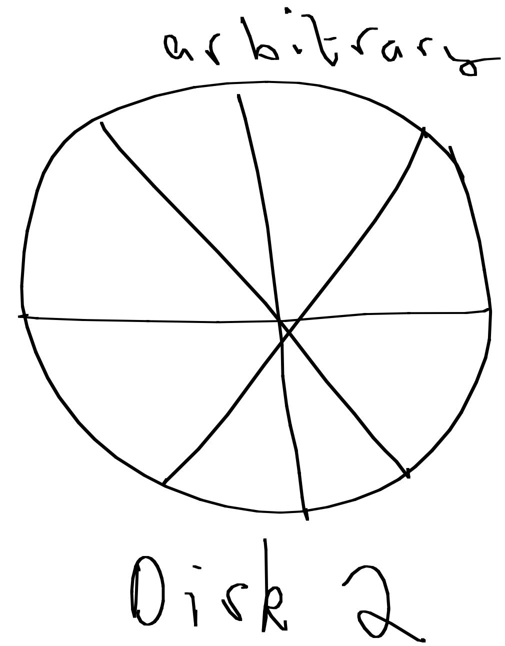
$|A| = n+1$

$\Rightarrow \exists\, i, j \in A$ s.t.

i & j are co-prime

$\exists\, i$ & j s.t. $j = i + 1$

$n+1$ number



200 Sectors on each

Place Disk 2 on top of Disk 1:

Claim: $\exists$ a placement such that at least 100 of Disk 2's sectors are on top of a Disk 1 sector of same color.

Proof 1

200 ways of putting Disk 2 onto Disk 1.

q_i = # of coincidences for position i.

$$q_1 + q_2 + \dots + q_{200} = 100 \times 200$$

Now apply PHP

j — Sector j of Disk 2

$$\begin{array}{cc} & j \\ \text{Position } i & \begin{bmatrix} & & \\ & 1_{j \text{ on top of same color}} & \\ & & \end{bmatrix} \leftarrow q_i \text{ ones} \\ & 100 \end{array}$$

200 x 200 matrix

Proof 2

Place Disk 2 randomly onto Disk 1.

$$X_i = \begin{cases} 1 & : \text{color same} \\ 0 & : \text{otherwise} \end{cases}$$

$$X = X_1 + \cdots + X_{200} = \#\text{ color same}$$

$$E(X) = \underbrace{E(X_1)}_{1/2} + \cdots + \underbrace{E(X_{200})}_{\frac{1}{2}}$$

$$= 100$$

Erdős - Szekerés

$a_1, a_2, \ldots, a_{k^2+1}$ are arbitrary real numbers.

Claim: $\exists$ a monotone subsequence of length $k+1$.

Monotone: $b_1, b_2, \ldots b_s$ is monotone if

$b_1 \leq b_2 \leq \cdots \leq b_s$ increasing

or $b_1 \geq b_2 \geq \cdots \geq b_s$ decreasing

$k=3$

Monotone increasing

$$27,\ 43,\ 12,\ 15,\ 10,\ 49,\ \ldots$$

Monotone decreasing.

Proof

$\ell = \ell(i)$

$a_i,\ a_i^1,\ a_i^2,\ \ldots\ a_i^{\ell-1}$ is a longest monotone increasing sequence starting at a_i.

a_2: 43, 49 $\quad \ell = 2$

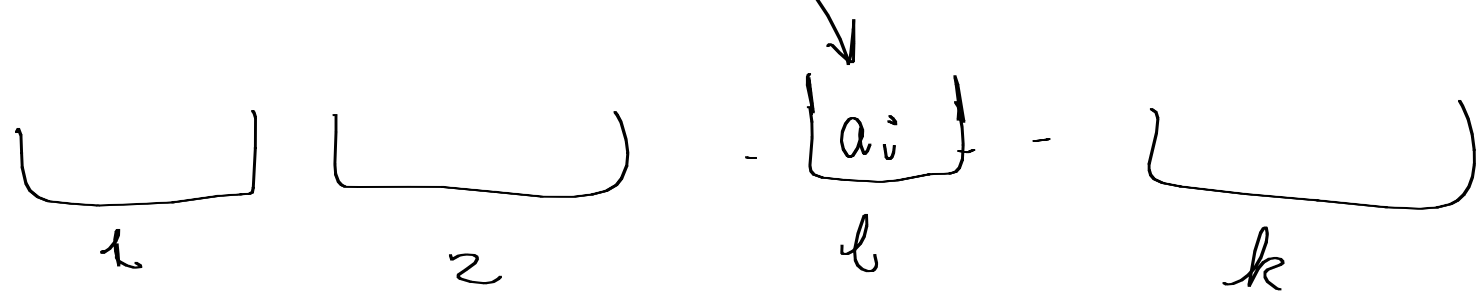
a_3: 12, 15, 49 $\quad \ell = 3$

Case 1: $\exists i$ s.t. $\ell(i) \geq k+1$

Nothing to do here.

Case 2: $\ell(a_i) \leq k$, $1 \leq i \leq k^2+1$

$\ell(a_i) = l$



$\exists$ box with $k+1$ element

$i_1 < i_2 < \cdots < i_{k+1}$

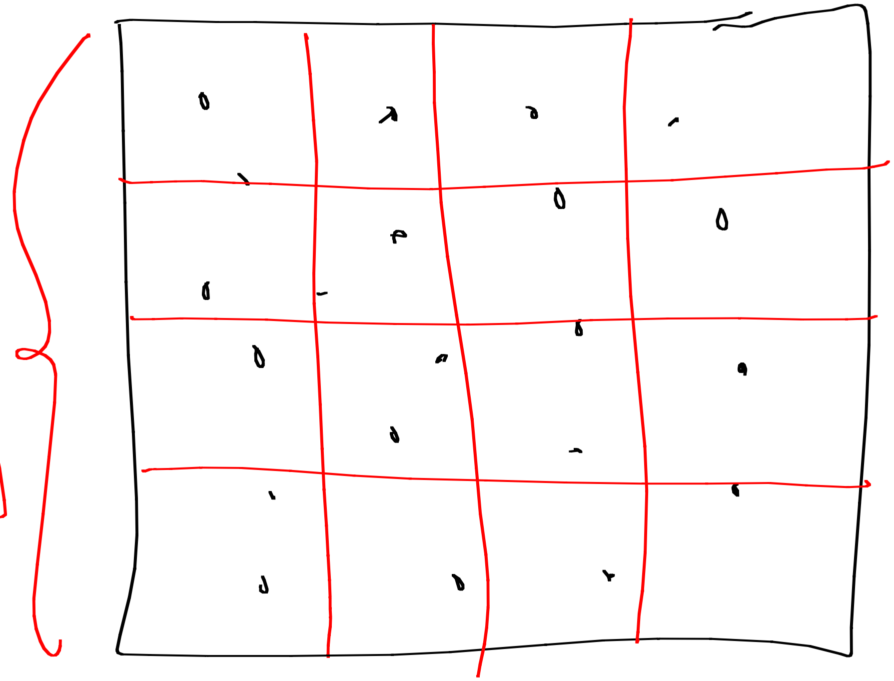
Claim: $a_{i_1} \geq a_{i_2} \geq \cdots \geq a_{i_{k+1}}$

If not:

$a_{i_j} < a_{i_{j+1}} \leq a^1_{i_{j+1}} \leq \cdots a^{\ell-1}_{i_{j+1}}$ ← NOT IN BOX

$\square$

$m = \left\lfloor \sqrt{\frac{n-1}{2}} \right\rfloor$



Put n points into unit square.

$\exists$ a triangle $P_i P_j P_k$ of small area.

$m^2 < \frac{n}{2}$

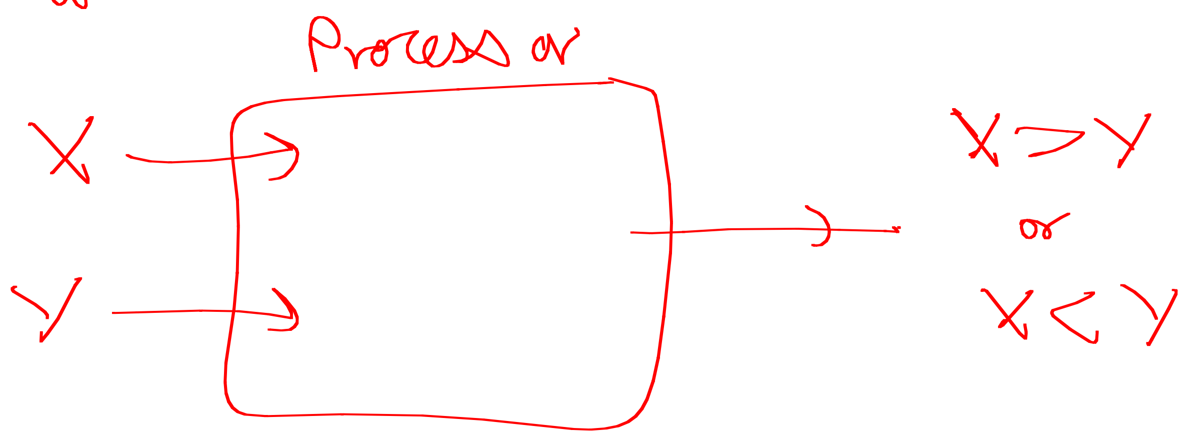
so $\exists$ $\square$ with ≥ 3 pts.

Parallel Search for Maximum

n processors

n numbers

In a round



Valiant: $\forall$ algorithms,

$\exists$ an ordering which requires $\geq \frac{1}{2}\log_2\log_2 n$ rounds.

<u>Set up</u>

Algorithm chooses n pairs to make comparisons.