11\28\2007

For any $x \in X$

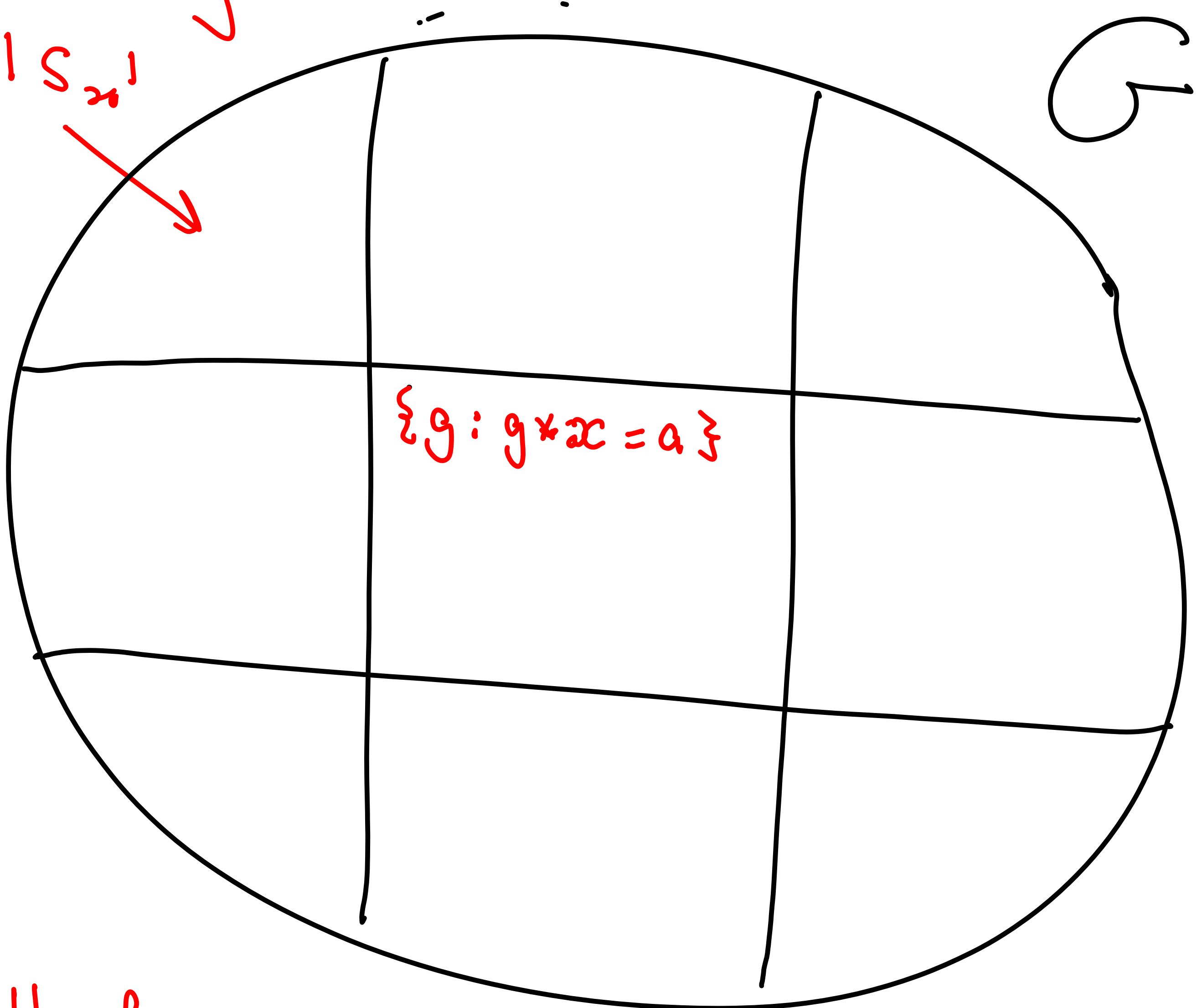
$$|O_x||S_x| = |G|.$$

Fix $x \in X$

Define equivalence relation $\sim$ on G

by $g \sim h$ iff $g * x = h * x$

Each class is of size $|S_x|$



\# of equivalence classes = \# a's = $|O_x|$

How many g's make $g * x = a$

Fix one of them g_1

$$g_1 * x = a$$

$$g_1 * x = a$$

$$x = g_1^{-1} * a$$

$$(g_1^{-1} \circ g_2) * x = g_1^{-1} * \underbrace{(g_2 * x)}_{a} = x$$

$$g_1^{-1} \circ g_2 \in S_x$$

$$g_2 \in g_1 \circ S_x$$

$\{g : g * x = a\}$

g_1 •

(i) $g_2 \in$ $\Rightarrow$ $g_2 \in g_1 \circ S_x$

(ii) Suppose $h = g_1 \circ s$ where $s \in S_x$

$h_1 * x = (g_1 \circ s) * x = g_1 * (s * x) = g_1 * x = a$

(iii) $s_1, s_2 \in S_x$ & $g_1 \circ s_1 = g_1 \circ s_2$

$\Rightarrow$ $s_1 = s_2$.

Theorem

$$|O_x||S_x| = |G|.$$

$$\#\text{orbits} = \sum_{x\in X} \frac{1}{|O_x|} = \sum_{x\in X} \frac{|S_x|}{|G|} = \frac{1}{|G|}\sum_{x\in X} |S_x|.$$

$\{1,2,3,\dots,10\}$

$= \{1,2,3\}$

$\cup \{4,5\}$

$\cup \{6,7,8,9\}$

$\cup \{10\}$

$\sum_{x\in X} =$

$\frac{1}{3}+\frac{1}{3}+\frac{1}{3}$

$+ \frac{1}{2}+\frac{1}{2} + \frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}$

$+1 = 4$

$$\mathrm{Fix}(g) = \{ x : g * x = x \}$$

$g \in G$.

$$\# \text{orbits} = \frac{1}{|G|} \sum_{g \in G} |\mathrm{Fix}(g)|$$

$$\begin{array}{cc} & g \\ x & \left[\begin{array}{c} \mathbb{1}_{g * x = x} \end{array} \right] \end{array} \quad \#1\text{'s in row } x = |S_x|$$

#1's in column $= |\mathrm{Fix}(g)|$