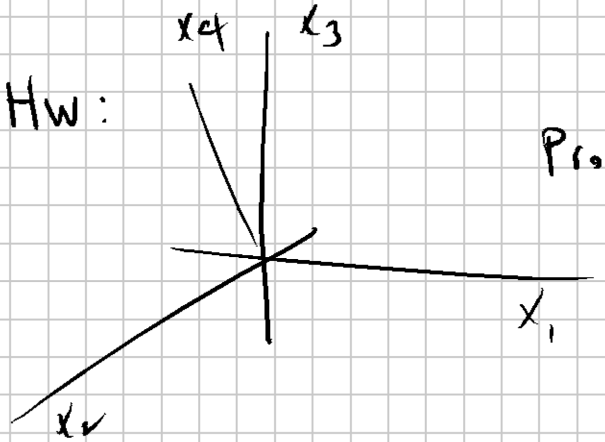
21-241 Lec 32 Matrix Algebra & Combinatorics!

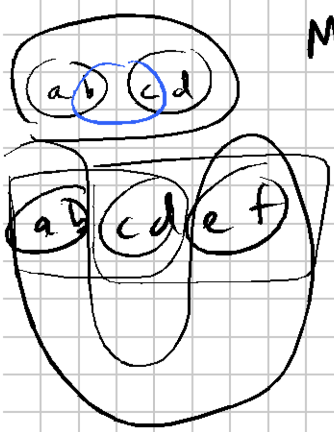
Note Title 11/17/2003

In Hw:



Project.

<u>Evens Town:</u> Population: 34

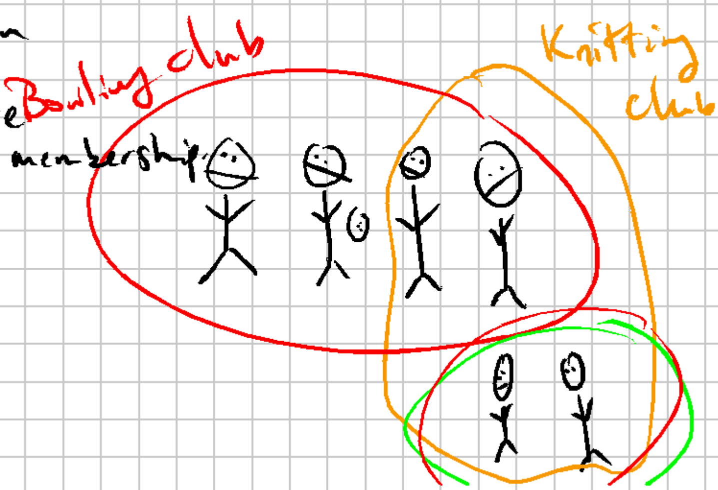


Make clubs:

a) Each club has even # of members

b) Each pair of clubs has even sized intersection

c) No clubs have <u>exactly</u> same membership.



How many clubs?

- 17 if pairs are each in 1 club

more?

Out of 34 people, make 17 pairs of people. "friends"

If person a is in club, then a's friend is also.

a b c d

1 0 0/1

$$2^{17} - 1 = 131{,}07\cancel{2}1 \text{ clubs}$$

a b c d e f

<u>Odds Town:</u>

a) Each club has an odd # of members

b) Each pair has an even # of common members

c) No duplicates.

How many clubs?

One club w/ 33 people, One club with other guy

At least 2 ...

<u>Groups of 3:</u> abc def

gets 12 clubs, but can't build...

a b c d

<u>Groups of 1:</u> a b c d ... e

gets 34 clubs.

<u>Groups of 33:</u> Club i has everyone but person i as members.

(Note: intersection size is 32 ← even)

also gets only 34 clubs.

Claim: Oddstown has at most 34 clubs.

Pf: Suppose the clubs are $C_1, C_2, \dots, C_m$.

Consider the "incidence" vector

$$\vec{v_i}(j) = \begin{cases} 0 & \text{if } j \notin C_i \\ 1 & \text{if } j \in C_i \end{cases}$$

(i.e. $\vec{v_i} = \begin{bmatrix} v_i(1) \\ v_i(2) \\ \vdots \\ v_i(34) \end{bmatrix}$)

$$\{\vec{v_1}, \vec{v_2}, \dots, \vec{v_m}\} \subseteq \mathbb{Z}_2^{34}$$

Lowest budget field (Integers mod 2)

in $\mathbb{Z}_2$: $1+1=0$ $\quad 0+0=0$

$0+1 = 1+0 = 1$

only numbers are $\{0,1\}$.

$1+1 \equiv 0 \pmod 2$

$\frac{1}{1}=1, \quad \frac{0}{1}=0.$

$\longrightarrow$ Division works right.

<u>Claim:</u> $\{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_m\}$ are linearly independent

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<u>Pf:</u> Suppose

$$d_1\vec{v}_1 + d_2\vec{v}_2 + \cdots + d_m\vec{v}_m = \vec{0}$$

then

$$\vec{v}_1 \cdot (d_1\vec{v}_1 + d_2\vec{v}_2 + \cdots + d_m\vec{v}_m) = \vec{v}_1 \cdot \vec{0} = 0.$$

$$\vec{v}_1 \cdot \vec{v}_1 \equiv |C_1| \pmod 2 \equiv 1 \pmod 2$$

$$\vec{v}_1 \cdot \vec{v}_2 \equiv |C_1 \cap C_2| \pmod 2 \equiv 0 \pmod 2$$

$$d_1(1) + d_2(0) + \cdots + d_m(0) = 0$$

$$d_1 = 0.$$

$$\begin{bmatrix}1\\1\end{bmatrix} \cdot \begin{bmatrix}1\\1\end{bmatrix} = 1+1 = 0$$

By same argument, $d_2 = \cdots = d_m = 0$.

$\{\vec{v}_1, \ldots, \vec{v}_m\}$ are lin. indep.

Span$\{\vec{v}_1, \ldots, \vec{v}_m\}$ is some m-dimensional space.

But $\mathbb{Z}_2^{34}$, which is 34-dim.

So $m \leq 34$.