

Final Exam Review Sheet

Exam 1 Type Stuff

1. Consider the linear System:

$$\begin{aligned}x_1 + 2x_2 + 3x_3 + 4x_4 &= 1 \\-x_2 - x_3 &= 0 \\x_1 + 3x_3 + 4x_4 &= 1\end{aligned}$$

a. Without doing any computation, show if this system has a solution then it is not unique.

b. Find all solutions.

2. Determine if $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \\ 5 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 4 \\ 5 \end{bmatrix} \right\}$ is linearly independent.

3. Find a linear transformation which rotates \mathbb{R}^2 90° CCW.

Post-Exam 3 type stuff

4. Let $f(x, y, z) = 2x^2 + y^2 + z^2 + 2xy$.

a. Write $f(x, y, z)$ as $[x \ y \ z] A \begin{bmatrix} x \\ y \\ z \end{bmatrix}$
where A is symmetric.

b. Find the orthogonal diagonalization of A .

c. Prove that $f(x, y, z) > 0$ for all $\begin{bmatrix} x \\ y \\ z \end{bmatrix} \in \mathbb{R}^3$.

d. Find $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ on the sphere of radius 1
so that $f\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right)$ is

i) maximized

ii) minimized