

Exam 3 Review Sheet

Show $\begin{bmatrix} 1 & 0 & -1 \\ 0 & 2 & -2 \\ 0 & 0 & 3 \end{bmatrix}$
and $\begin{bmatrix} 3 & 1 & 1 \\ -2 & 0 & -3 \\ 0 & 0 & 3 \end{bmatrix}$ are similar.

Find the eigenvalues & corresponding eigenspaces of $\begin{bmatrix} 4 & 3 \\ -3 & 4 \end{bmatrix}$.

Is $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ diagonalizable?

Find $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^n$

$A = \begin{bmatrix} 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 \end{bmatrix}$. Find an orthonormal basis for row A .

Let U be orthonormal $n \times n$ matrix. Show $U^T = U^{-1}$

Find the eigenvalues and their geometric multiplicities in

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 & 2 \\ 3 & 3 & 3 & 3 & 3 \\ 4 & 4 & 4 & 4 & 4 \\ 5 & 5 & 5 & 5 & 5 \end{bmatrix}$$

Suppose A has eigenvalues $\lambda_1, \lambda_2, \dots, \lambda_n$.

Show A^{-1} has eigenvalues $\frac{1}{\lambda_1}, \frac{1}{\lambda_2}, \dots, \frac{1}{\lambda_n}$.

What are the eigenvalues of $A - I + (A - I)^{-1}$?

$$\vec{a} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{bmatrix}, \vec{b} = \begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \\ 1 \end{bmatrix}, \vec{c} = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \\ 0 \end{bmatrix}$$

Find $\vec{x} \in \text{span}\{\vec{b}, \vec{c}\}$ to minimize $\|\vec{a} - \vec{x}\|$.

Find an orthogonal set
of 3 vectors containing

$$\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}.$$

What is the span of your set?

Let A be an $m \times n$ matrix,
and let $\vec{x} \in \mathbb{R}^n$.

Show $\vec{x}^T A^T A \vec{x} \geq 0$.

Let \vec{x} & \vec{y} be n -
dimensional vectors.

Let $L = \text{span}\{\vec{y}\}$.

Show that if

$$\vec{x} = \vec{x}_1 + \vec{x}_2$$

where $\vec{x}_1 \cdot \vec{x}_2 = 0$

and $\vec{x}_2 \in L^\perp$

then $\vec{x}_1 = \frac{\vec{x} \cdot \vec{y}}{\vec{y} \cdot \vec{y}} \vec{y}$

Let A be an $m \times n$ orthonormal matrix.

Show that $\dim(\text{col } A) = n$