21-241 Sec C Exam 2 Review Sheet

Find a linear transform which rotates the plane $45°$ CCW around the point $(51,-7)$.
Express your answer as a single 3×3 matrix

Find $\begin{bmatrix}0&1\\1&0\end{bmatrix}^{-1}$, $\begin{bmatrix}0&1&1\\1&0&1\\1&1&0\end{bmatrix}^{-1}$,

$$\begin{bmatrix}0&1&\cdots&1&1\\1&0&\cdots&1&1\\\vdots&\vdots&&\vdots&\vdots\\1&1&\cdots&1&0\end{bmatrix}^{-1}$$
(n cols)

$A = \begin{pmatrix}3&3&3&1\\3&1&0&3\\-3&0&1&3\\0&3&3&1\end{pmatrix}$. Find A^{-1}.

A, B $n\times n$ matricies.
~~Show that~~ Prove: If AB is invertible then A is invertible

Find a 2×2 matrix with column space of dimension 1.
Sketch null space + column space.

Find a basis for null space and column space of

$$\begin{bmatrix}1&1&1&1\\1&1&-1&-1\\1&-1&-1&1\\-1&1&1&-1\end{bmatrix}.$$

Is there a basis for the column space containing $\begin{bmatrix}2\\-2\\2\\-2\end{bmatrix}$?

Give an example of 2×2 matricies $A \neq B$ where $A^2-B^2 \neq (A+B)(A-B)$.

A is a 2×3 matrix with $\text{nul}A = \mathbb{R}^3$.
Find $\text{col}A$. Find an A like this.

A is a 3×3 matrix where with $\text{col}A = \mathbb{R}^3$.
Find $\text{nul}A$

Consider the matrix A on the left a corresponding echelon form is given on the right.

$$\begin{bmatrix} -3 & 9 & -2 & -7 \\ 2 & -6 & 4 & 8 \\ 3 & -9 & -2 & 2 \end{bmatrix} \to \begin{bmatrix} 1 & -3 & 6 & 9 \\ 0 & 0 & 4 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Does the pair

$$\begin{bmatrix} 9 \\ -6 \\ -9 \end{bmatrix}, \begin{bmatrix} -7 \\ 8 \\ 2 \end{bmatrix}$$

form a basis for the column space of A? Explain.

Find the determinants:

$$\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}, \quad \begin{vmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix}, \quad \begin{vmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{vmatrix}$$

A nxn matrix with $\det A = 3$. Find $\det(A^3 A^T A^{-1})$.

Find a 3x3 matrix with eigenvalues 3 and 1. What are some corresponding eigenvectors?

Show 3 is an eigenvalue of

$$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}.$$

Show $\begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$ is an eigenvector