Review Problems for Test 4

1. Assume that $\int_0^5 f(x)dx = -7$, $\int_0^1 f(x)dx = 2$, and $\int_1^5 g(x)dx = 6$.

   (a) Find $\int_1^5 f(x)dx$.

   (b) Find $\int_1^5 (3f(x) - 4g(x))dx$.

2. Determine each of the following:

   (a) $\dfrac{d}{dx}\displaystyle\int_{lnx}^{x^2} \sqrt{1+t^4}\ dt$

   (b) $\dfrac{d}{dx}\displaystyle\int_{x}^{3} e^{t^3}\ dt$

3. Evaluate each of the following indefinite integrals

   (a) $\int (x^2 + x^3 - 3x^4)\, dx$

   (b) $\int (e^{3x} + e^{-2x})\, dx$

   (c) $\int \sqrt{x}(x+1)^2 dx$

   (d) $\int e^{-x}\,(e^{2x} + e^{5x})\, dx$

4. Evaluate each of the following definite integrals

   (a) $\int_0^1 (x^3 - 6x^2 + 1)dx$

   (b) $\displaystyle\int_4^9 \frac{dz}{\sqrt{z}}$

   (c) $\displaystyle\int_0^{ln 3} \left(\frac{e^x + e^{3x}}{e^{2x}}\right) dx$

5. Evaluate each of the following indefinite integrals.

   (a) $\displaystyle\int \frac{x^3}{(1-x^4)^6}\ dx$

(b) $\displaystyle\int \frac{x^2}{1+x^3}\, dx$

(c) $\displaystyle\int \frac{dx}{x\ lnx}$

(d) $\displaystyle\int x^3 lnx dx$

(e) $\displaystyle\int x^2\sqrt{x-1}\, dx$

(f) $\displaystyle\int \sqrt{1+\sqrt{x}}\, dx$

6. Evaluate each of the following definite integrals

(a) $\displaystyle\int_0^6 \sqrt{2x+4}\, dx$

(b) $\displaystyle\int_2^3 \frac{dx}{x(lnx)^2}$

(c) $\displaystyle\int_0^1 xe^{2x} dx$

(d) $\displaystyle\int_0^1 \frac{dx}{1+\sqrt{x}}$

7. Given that f satisfied $f'(x) = f(x)^2$ for all $x \in [0,1]$ and $f(0) = -1,\ f(1) = -\dfrac{1}{2}$, find $\displaystyle\int_0^1 f(x)^4 dx$.

8. Evaluate each limit by identifying it with a definite integral.

(a) $\displaystyle\lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} e^{\frac{i}{n}}$

(b) $\displaystyle\lim_{n\to\infty} \frac{3}{n} \sum_{i=1}^{n} \sqrt{1+\frac{3i}{n}}$

9. Find the area of

(a) the region bounded by $y = x+1$ and $y = x^2 - 1$.

(b) the region bounded by $y = 0,\ y = lnx$, and $x = e$.

10. Find the volume generated when

(a) the region bounded by $y = 0,\ y = e^{3x},\ x = 0$, and $x = 1$ is rotated about the x-axis.

(b) the region bounded by $y = \sqrt{x},\ y = 2$, and $x = 0$ is rotated about the y-axis.