Department of Mathematical Sciences
Carnegie Mellon University

21-476 Ordinary Differential Equations Fall 2003

Assignment 9

1. For each choice of A given below, determine the stability of the zero solution of $\dot{x} = Ax$.

a) $A = \begin{pmatrix} 0 & 1 \\ 4 & 0 \end{pmatrix}$ b) $A = \begin{pmatrix} 1 & 5 \\ -1 & 3 \end{pmatrix}$

c) $A = \begin{pmatrix} -1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$

1. For each autonomous system and corresponding critical point x^* given below, use the method of linearization to deduce as much as you can about the stability of x^*.

a)
$$\begin{aligned} \dot{x}_1 &= 3x_1 - x_2 + x_3^3 \\ \dot{x}_2 &= x_1 - x_2 + x_2x_3 \\ \dot{x}_3 &= 2x_1 + x_1x_2 - x_3 + x_3^2 \end{aligned} \qquad x^* = (0,0,0)$$

b)
$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -x_1 - x_1^2 \end{aligned} \qquad x^* = (-1,0)$$

c)
$$\begin{aligned} \dot{x}_1 &= x_2 - x_2^3 \\ \dot{x}_2 &= x_1 \end{aligned} \qquad x^* = (0,1)$$

d)
$$\begin{aligned} \dot{x}_1 &= -2\sin x_1 + x_1^2 - x_2 \\ \dot{x}_2 &= x_1\cos x_1 - 2x_2 + x_2^3 \end{aligned} \qquad x^* = (0,0)$$