Supplementary Problems for Assignment 7

1. Determinant Tic-Tac-Toe: This is a game involving two players, labelled 0 and 1. Player 1 starts by putting a 1 in some location in a 3×3 matrix. Player 0 then places a zero in different location. The players alternate turns until the matrix is filled in with five 1's and four 0's. Player 0 wins if the determinant is 0; otherwise player 1 wins. Does either player having a winning strategy? If so, give such a strategy.

2. Prove or Disprove: Let V be a finite dimensional vector space over $\mathbb{R}$ with inner product $(\cdot, \cdot)$. Let $T \in L(V, V)$ be given and assume that

$$(Tx, y) = -(x, Ty)$$

for all $x, y \in V$. Then $\det(T) = 0$.

3. Let $\mathbb{F} = \mathbb{Z}_3$. Compute $\det(A)$ and $\det(B)$, where

$$A = \begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 1 & 0 & 2 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 1 & 0 & 1 \\ 2 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 2 & 1 & 2 \end{pmatrix}.$$

4. Let $\mathbb{F} = \mathbb{R}$. Let A be a 3×3 matrix all of whose entries are 1 or -1, and B be a 3×3 matrix all of whose entries are 0 or 1.

   (a) What is the largest possible value for $\det(A)$?

   (b) What is the largest possible value for $\det(B)$?