Due on Friday, September 5

In Problems 1-4, solve the given differential equation or initial value problem.

1. $\dfrac{dx}{dt} = 3x - e^t;\ x(0) = -2.$

2. $\dot{x}(t) = \dfrac{-\left(4x(t)^5 t^3 + \cos t\right)}{5x(t)^4 t^4 + 1}.$

3. $\dot{x}(t) + 2t x(t) = t^3;\ x(0) = 1.$

4. $\dfrac{dx}{dt} = e^x e^{2t} - 1.$

5. Solve the integral equation

$$x(t) = 1 + \int_0^t e^{-(t-\tau)} \left[x(\tau) + x(\tau)^2\right] d\tau.$$

6. Let I be an interval and $\alpha \in \mathbb{R}\backslash\{0, 1\}$ be given. Consider the differential equation

$$(*) \qquad \dot{x}(t) + p(t)x(t) = q(t)x(t)^{\alpha},$$

where $p, q : I \to \mathbb{R}$ are given continuous functions. Let x be a strictly positive solution of $(*)$ on some interval $J \subset I$ and put $y(t) = x(t)^{1-\alpha}$ for all $t \in J$. Find a first-order linear differential equation satisfied by y.

7. Determine as much as you can about the solution of

$$\dot{x}(t) = t^2 + x(t)^2;\ x(0) = 0.$$