21-342 **Linear Algebra II** **Spring 2004**

Supplementary Problems for Assignment 3

1. * Let $n \in \mathbb{Z}^+$ with $n \geq 2$ be given. Let $\mathbb{F} = \mathbb{R}$ and

$$V = V' = \{f \in P_n(\mathbb{R}) : f(0) = f(1) = 0\}$$

and define $T \in L(V, V)$ by

$$(Tf)(t) = t(t-1)\ddot{f}(t) \quad \forall t \in \mathbb{R},\ f \in V.$$

(Here $P_n(\mathbb{R})$ is the set of all real polynomials of degree $\leq n$ and $\ddot{f}$ is the second derivative of f.)

Consider the bilinear form $B : V \times V' \to \mathbb{R}$ defined by

$$B(f, g) = \int_0^1 f(t)g(t)dt \quad \forall f \in V,\ g \in V'.$$

(You may take it for granted that B is nondegenerate.)

Find the transpose $T' \in L(V', V')$ of T with respect to B.

2. * Let $q \in P_{99}(\mathbb{R})$ be given. Show that there exists $Q \in P_{99}(\mathbb{R})$ such that

$$\int_0^1 q(x)f(x)dx = \int_{-\pi}^{\pi} e^x(\sin^2 x)Q(x)f(x)dx \quad \text{for all } f \in P_{qq}(\mathbb{R}).$$