Exam 1 Review

May 29, 2014

1 Technique Question

The following is a list of question. Contained in this list are some that will be chosen to put on the test. Note that none of these proofs needs to be particularly long. Some can be done with two sentences. Some can be done with just a string of equalities and algebraic properties we proved in class.

1. Let $\mathbf{u}, \mathbf{v} \in \mathbb{R}^n$. Prove that
$$\operatorname{span}(\mathbf{u}+\mathbf{v}, \mathbf{u}) = \operatorname{span}(\mathbf{u}, \mathbf{v})$$
2. Prove that any subset of a linearly independent set is linearly independent.
3. Prove that if $\{\mathbf{u}_1, \ldots, \mathbf{u}_m\}$ is linearly dependent then $\{\mathbf{u}_1, \ldots, \mathbf{u}_m, \mathbf{u}\}$ is as well.
4. Prove that $A^T A$ is symmetric.
5. Let A be an invertible $n \times n$ matrix. Prove that the rows and columns are linearly independent.
6. Let A and B be square matrices of the same dimensions. Prove that if $AB = I$ then $BA = I$ (note: you are not assuming anything about the invertibility of A or B).
7. Let A and B be diagonal matrices of the same size. Prove that $AB = BA$.
8. We define the trace of a $n \times n$ matrix A by
$$\operatorname{tr}(A) = \sum_{i=1}^{n} a_{ii}$$
That is, the trace is the sum along the diagonal. Prove that $\operatorname{tr}(A+B) = \operatorname{tr}(A) + \operatorname{tr}(B)$.
9. We define the trace of a $n \times n$ matrix A by
$$\operatorname{tr}(A) = \sum_{i=1}^{n} a_{ii}$$
That is, the trace is the sum along the diagonal. Prove that $\operatorname{tr}(AB) = \operatorname{tr}(BA)$.
10. Prove that if A and B are invertible then AB is invertible, and find the inverse.
11. Prove that if A if an invertible $n \times n$ matrix and c nonzero then cA is invertible. Find it's inverse.
12. Prove that the inverse of a symmetric matrix is symmetric.
13. Let A be a diagonal matrix. Prove A is invertible if and only if no element on the diagonal is nonzero.

2 Other Review

2.1 Definitions

Be sure that you can define all of the important words that we defined in class. If you look through my notes, they are in bold. Here is a sampling (I'm not precisely sure it is complete).

> Gauss Jordan Elimination, Gaussian Elimination, R^n, associativity, augmented matrix, column vector, commutativity, dot product, elementary matrix, elementary row operations, homogeneous equation, identity matrix, inverse, leading entry, line, linear, linear combination, linear equation, linearly dependent, linearly independent, m by n matrix, matrix, norm, plane, rank, reduced row echelon form, row echelon form, row vector, scalar matrix, solution, solution set, span, square matrix, symmetric matrix, system of linear equations, transpose, trivial solution, vector, zero vector

2.2 Major Theorems

Be sure that you know all the major theorems that we proved. A component of the test will be able to state/use easy corollaries to theorems in a multiple choice or true/false manner. For example, a question on the test might be:

> True/False: The rank of a matrix is the number of nonzero rows that it has in row echelon form.

This is a direct application of the theorems that we did involving exploring rank using row vectors and elementary row operations.

2.3 Major Computational Algorithms

We've done a few computational algorithms. They all involve row reduction of a (possibly augmented) matrix. They were all for different, but relating, purposes, including:

- Solving a system of equation
- Determining the span of a set of vectors
- Determining if a set of vectors is linearly independent or dependent.
- Determining and finding the inverse of a matrix.

It is important that you are proficient in row operations but it is much more important that you are proficient at interpreting results. So if I give you a matrix in (reduced) row echelon form, you should be able to say a lot about the columns/rows/system/matrix.