① $\int_1^2 \frac{(x+1)^2}{x}\,dx = \int_1^2 \frac{x^2+2x+1}{x}\,dx = \int_1^2 \left(x+2+\frac{1}{x}\right)dx$

$= \left[\frac{1}{2}x^2+2x+\ln|x|\right]_1^2$

② $\int_1^2 \frac{x}{(x+1)^2}\,dx$ $\quad u = x+1$, $du = dx$ $\quad = \int_2^3 \frac{u-1}{u^2}\,du = \int_2^3 \frac{1}{u} - \frac{1}{u^2}$

$= \left[\ln|u| + \frac{1}{u}\right]_2^3$

③ $\int_0^{\pi/2} \sin\theta\, e^{\cos\theta}\,d\theta$ $\quad u = \cos\theta$, $du = \sin\theta\, d\theta$ $\quad = -\int_1^0 e^u\,du = \left[-e^u\right]_1^0$

④ $\int_0^{\pi/6} t\sin(2t)\,dt$ $\quad u = 2t$, $du = 2dt$ $\quad = \frac{1}{2}\int_0^{\pi/3} \frac{1}{2}u\sin(u)\,du = \frac{1}{4}\int_0^{\pi/3} u\sin(u)\,du$

$w = u$, $dw = du$ $\quad dv = \sin(u)\,du$, $V = -\cos(u)$ $\quad = \frac{1}{4}\left[-u\cos(u)\Big|_0^{\pi/3} + \int_0^{\pi/3}\cos(u)\,du\right]$

$= \frac{1}{4}\left[-u\cos(u) + \sin(u)\right]_0^{\pi/3}$

⑤ $\int \frac{dt}{2t^2+3t+1} = \int \frac{dt}{(2t+1)(t+1)}$

$b^2-4ac = 9-4(2)(1)$
$= 1$

$\frac{1}{(2t+1)(t+1)} = \frac{A}{2t+1} + \frac{B}{t+1}$

$1 = A(t+1) + B(2t+1)$

@ $t=-1$, $1 = B(-1) \Rightarrow B = -1$.

@ $t=-\frac{1}{2}$ $\quad 1 = \frac{1}{2}A \Rightarrow A = 2$

$= \int\left(\frac{2}{2t+1} + \frac{-1}{t+1}\right)dt$

$= \frac{2}{2}\ln|2t+1| - \ln|t+1| + C$

⑥ $\int_1^2 x^5 \ln(x)\, dx$ $\quad u = \ln(x) \quad dv = x^5 dx$
$\quad du = 1/x\, dx \quad v = \frac{1}{6}x^6$

$$= \frac{1}{6}x^6 \ln(x)\Big|_1^2 - \int_1^2 \frac{1}{6}x^6 \frac{1}{x}\,dx = \frac{1}{6}x^6 \ln(x)\Big|_1^2 - \frac{1}{6}\int_1^2 x^5\,dx$$

$$= \left[\frac{1}{6}x^6 \ln(x) - \frac{1}{6}\cdot\frac{1}{6}x^6\right]_1^2$$

⑦ $\int_0^{\pi/2} \sin^3(\theta)\cos^2(\theta)\,d\theta$ $\quad u = \cos\theta \quad du = -\sin\theta\, d\theta$ $\quad = \int_0^{\pi/2} \sin\theta\,(1-\cos^2\theta)\cos^2\theta\, d\theta$

$$= -\int_1^0 (1-u^2)u^2\, d\theta = \int_0^1 (u^2 - u^4)\,du = \left[\frac{u^3}{3} - \frac{u^5}{5}\right]_0^1$$

⑧ $\int \frac{dx}{\sqrt{e^x - 1}}$ $\quad u = e^x \quad du = e^x dx$ $\quad = \int \frac{e^x dx}{e^x\sqrt{e^x-1}}$

$$= \int \frac{du}{u\sqrt{u-1}} \qquad w = u-1 \quad dw = du$$

$$= \int \frac{dw}{(w+1)\sqrt{w}} \qquad v = \sqrt{w} \quad dv = \frac{1}{2\sqrt{w}}\,dw$$

$$= \int \frac{2\,dv}{v^2+1} = 2\tan^{-1}(v)$$

$$= 2\tan^{-1}(\sqrt{w})$$

$$= 2\tan^{-1}(\sqrt{u-1})$$

$$= 2\tan^{-1}(\sqrt{e^x-1}) + C$$

(9) $\displaystyle\int \frac{\sin(\ln(t))}{t}\,dt$ $\quad u = \ln(t)$, $du = \frac{1}{t}\,dt$

$$= \int \sin(u)\,du$$

$$= -\cos(u) + C$$

$$= -\cos(\ln(t)) + C$$

(10) $\displaystyle\int_0^1 \frac{\sqrt{\tan^{-1}(x)}}{1+x^2}\,dx$ $\quad \left(\begin{array}{l} u = \tan^{-1}(x) \\ du = \frac{1}{1+x^2}\,dx \end{array}\right)$

$$= \int_0^{\pi/4} \sqrt{u}\,du$$

$$= \left[\frac{2}{3}u^{3/2}\right]_0^{\pi/4}$$

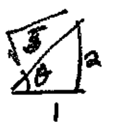
(4) $\displaystyle\int_1^2 \frac{\sqrt{x^2+1}}{x}\,dx$ $\quad \left(\begin{array}{l} x = \tan\theta \\ dx = \sec^2\theta\,d\theta \end{array}\right)$

$$= \int_{\pi/4}^{\arctan(2)} \frac{(\sec\theta)(\sec^2\theta)}{\tan\theta}\,d\theta$$

$$= \int_{\pi/4}^{\arctan(2)} \frac{1}{\cos^3\theta}\cdot\frac{\cos\theta}{\sin\theta}\,d\theta = \int_{\pi/4}^{\arctan(2)} \frac{\cos\theta\,\sin\theta}{\cos^3\theta\,\sin^2\theta}\,d\theta$$

$u = \cos\theta$
$du = -\sin\theta\,d\theta$

$\cos(\arctan(2)) = \frac{1}{\sqrt{5}}$



$$= \int_{1/\sqrt{2}}^{\boxed{1/\sqrt{5}}} \frac{-u}{u^3(1-u^2)}\,du$$

$$\frac{-u}{u^3(1-u)(1+u)} = \frac{A}{u} + \frac{B}{u^2} + \frac{C}{u^3} + \frac{D}{1-u} + \frac{E}{1+u}$$

(I should have cancelled a "u" first here. I made it more complicated. My bad!)

$-u =$ $Au^2(1-u)(1+u) + Bu(1-u)(1+u) + C(1-u)(1+u)$
$\quad + Du^3(1+u) + Eu^3(1-u)$

$@u=0$ $\boxed{C=0}$ $\qquad @u=-1$ $\quad 1 = E(-1)(2)$

$@u=1$ $\boxed{D=-\frac{1}{2}}$ $\qquad \boxed{E=-\frac{1}{2}}$

$u^4[-A + \underbrace{D - E}_{0}]$ $\quad \boxed{A=0}$

$u^3[-B + D + E]$ $\quad \boxed{B=1}$

$$= \int_{1/\sqrt{2}}^{1/\sqrt{5}} \left(\frac{1}{u^2} - \frac{1/2}{1-u} - \frac{1/2}{1+u} \right) du$$

$$= \left[-\frac{1}{u} + \frac{1}{2}\ln|1-u| - \frac{1}{2}\ln|1+u| \right]_{1/\sqrt{2}}^{1/\sqrt{5}}$$

(Maybe there is an easier way?)

(12) $\displaystyle\int \frac{e^{2x}}{1+\underbrace{e^{4x}}_{(e^{2x})^2}}\,dx$ $\quad u = e^{2x}$, $\quad du = 2e^{2x}dx$ $\quad = \frac{1}{2}\int \frac{du}{1+u^2}$

$$= \frac{1}{2}\tan^{-1}(e^{2x}) + C$$

(13) $\displaystyle\int e^{\sqrt[3]{x}}\,dx$ $\quad u = \sqrt[3]{x}$

$du = \frac{1}{3x^{2/3}}dx$

$= \frac{1}{3u^2}dx$ $\quad = 3\int u^2 e^u\,du$

$w = u^2 \qquad dv = e^u du$

$dw = 2u \qquad v = e^u$

$$= 3\left[u^2e^u - 2\int ue^u du\right]$$

$w = u \qquad dv = e^u du$

$dw = du \qquad v = e^u$

$$= 3u^2e^u - 6ue^u + 6\int e^u du$$

$$= 3u^2e^u - 6ue^u + 6e^u + C$$

$$= 3x^{2/3}e^{\sqrt[3]{x}} - 6\sqrt[3]{x}e^{\sqrt[3]{x}} + 6e^{\sqrt[3]{x}} + C$$

(14) $\int \frac{x^2+2}{x+2}\,dx \quad \begin{pmatrix} u = x+2 \\ du = dx \end{pmatrix} = \int \frac{(u-2)^2+2}{u}\,du$

$$= \int \frac{u^2-4u+4+2}{u}\,du = \int u - 4 + \frac{6}{u}\,du$$

$$= \frac{1}{2}u^2 - 4u + 6\ln|u| = \frac{1}{2}(x+2)^2 - 4(x+2) + 6\ln|x+2| + C$$

(15) $\int \frac{x-1}{x^2+2x}\,dx \qquad \frac{x-1}{x(x+2)} = \frac{A}{x} + \frac{B}{x+2}$

$$x-1 = A(x+2) + Bx$$

$$= x(A+B) + 2A$$

$$A = -\frac{1}{2} \qquad B = \frac{3}{2}.$$

$$= \int\left(\frac{-1/2}{x} + \frac{3/2}{x+2}\right)dx = -\frac{1}{2}\ln|x| + \frac{3}{2}\ln|x+2| + C$$

(16) $\int \frac{\sec^6\theta}{\tan^2\theta}\,d\theta \begin{pmatrix} u = \tan\theta \\ du = \sec^2\theta\,d\theta \end{pmatrix} = \int \frac{(1+\tan^2\theta)^2}{\tan^2\theta}\sec^2\theta\,d\theta$

$$= \int \frac{(1+u^2)^2}{u^2}\,du$$

$$= \int \frac{u^4+2u^2+1}{u^2}\,du$$

$$= \int\left(u^2+2+\frac{1}{u^2}\right)du$$

$$= \frac{1}{3}u^3 + 2u - \frac{1}{u} + C$$

$$= \frac{1}{3}\tan^3\theta + 2\tan\theta - \frac{1}{\tan\theta} + C$$

(17) $\int x \sec x \tan x \, dx$

$u = x \qquad dv = \sec x \tan x \, dx$

$du = dx \qquad v = \sec x$

$= x\sec(x) - \int \sec(x)\,dx$

$= x\sec(x) - \ln|\sec(x) + \tan(x)| + C$

(18) $\int \frac{x^2+8x-3}{x^3+3x^2}\,dx \qquad u = x^3+3x^2$

$du = (3x^2+6x)\,dx$

$= \frac{1}{3}\int \frac{3x^2+24x-9}{x^3+3x^2}\,dx = \frac{1}{3}\int \frac{3x^2+6x}{x^3+3x^2}\,dx + \frac{1}{3}\int \frac{18x-9}{x^3+3x^2}\,dx$

$= \frac{1}{3}\ln|x^3+3x^2| + \int \frac{6x-3}{x^3+3x^2}\,dx$

$\frac{6x-3}{x^2(x+3)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+3}$

$(6x-3) = Ax(x+3) + B(x+3) + Cx^2$

$@x=0 \quad \boxed{B=-1}$

$@x=-3 \quad -21 = 9C$

$C = \frac{-21}{9} = -\frac{7}{3}$

$x^2(A+C) \qquad \boxed{A = \frac{7}{3}}$

$= \boxed{\frac{1}{3}\ln|x^3+3x^2| + \frac{7}{3}\ln|x| - \frac{7}{3}\ln|x+3| + \frac{1}{x} + C}$

(19) $\displaystyle\int \frac{x+1}{9x^2+6x+5}\,dx = \frac{1}{9}\int \frac{x+1}{x^2+\frac{2}{3}x+\frac{5}{9}}$

$b^2-4ac = 36 - 4(9)(5) < 0$

$u = x^2+\frac{2}{3}x+\frac{5}{9}$

$du = \left(2x+\frac{2}{3}\right)dx$

$$= \frac{1}{18}\int \frac{2x+2}{x^2+\frac{2}{3}x+\frac{5}{9}}\,dx$$

$$= \frac{1}{18}\left[\int \frac{2x+\frac{2}{3}}{x^2+\frac{2}{3}x+\frac{5}{9}}\,dx + \int \frac{4/3}{x^2+\frac{2}{3}x+\frac{5}{9}}\,dx\right]$$

$$= \frac{1}{18}\left[\ln\left|x^2+\frac{2}{3}x+\frac{5}{9}\right| + \quad '' \quad\right]$$

$$= \frac{1}{18}\left[\quad '' \quad + \frac{4}{3}\int \frac{dx}{\left(x^2+\frac{2}{3}x+\frac{1}{9}\right)+\frac{4}{9}}\,dx\right.$$

$$= \frac{1}{18}\left[\quad '' \quad + \frac{4}{3}\int \frac{dx}{\left(x+\frac{1}{3}\right)^2+\frac{4}{9}}\right.$$

$$= \frac{1}{18}\left[\ln\left|x^2+\frac{2}{3}x+\frac{5}{9}\right| + \frac{4}{3}\cdot\frac{3}{2}\tan^{-1}\left(\frac{3\left(x+\frac{1}{3}\right)}{2}\right)\right] + C$$

(20) $\int \tan^5\theta \sec^3\theta \, d\theta = \int \tan^4\theta \sec^2\theta \quad \sec\theta\tan\theta \, d\theta$

$u = \sec\theta \, d\theta$

$$= \int (u^2-1)^2 u^2 \, du$$

$$= \int (u^6 - 2u^4 + u^2) \, du$$

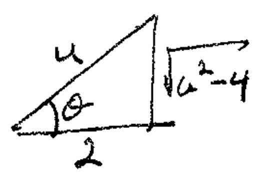
$$\frac{1}{7}\sec^7(\theta) - \frac{2}{5}\sec^5(\theta) + \frac{1}{3}\sec^3(\theta) + C$$

(21) $\int \frac{dx}{\sqrt{x^2-4x}} = \int \frac{dx}{\sqrt{x}\sqrt{x-4}}$ $\quad u = \sqrt{x}$, $du = \frac{1}{2\sqrt{x}}dx$ $\quad = 2\int \frac{du}{\sqrt{u^2-4}}$

$u = 2\sec\theta$

$du = 2\sec\theta\tan\theta \, d\theta$

$$= 2\int \frac{\cancel{2\sec\theta\tan\theta} \, d\theta}{\cancel{2\tan\theta}}$$



$$= 2\ln|\sec\theta + \tan\theta| + C$$

$$= 2\ln\left|\frac{u}{2} + \frac{\sqrt{u^2-4}}{2}\right| + C_1$$

$$= 2\ln\left|\frac{\sqrt{x}}{2} + \frac{\sqrt{x-4}}{2}\right| + C_1$$

(22) $\int t e^{\sqrt{t}}\, dt$ $\quad u = \sqrt{t}$, $\quad du = \frac{1}{2\sqrt{t}}\, dt$ $\quad = 2\int \frac{t^{3/2} e^{t^{1/2}}}{2t^{1/2}}\, dt$

$$= 2\int u^3 e^u \, du$$

Do IBP 3 times

(see prob 13 for similar)

(23) $\int \frac{dx}{x\sqrt{x^2+1}}$ $\quad \begin{pmatrix} u = x^2+1 \\ du = 2x\,dx \end{pmatrix}$ $\quad = \frac{1}{2}\int \frac{2x\,dx}{x^2\sqrt{x^2+1}}$

$$= \frac{1}{2}\int \frac{du}{(u-1)\sqrt{u}}$$

$w = \sqrt{u}$

$dw = \frac{1}{2\sqrt{u}}\, du$ $\quad = \frac{\cancel{2}}{\cancel{2}}\int \frac{1}{w^2-1}\, dw$

$w = \sec\theta$

$du = \sec\theta \tan\theta \, d\theta$

$$= \int \frac{\sec\theta \cancel{\tan\theta}\, d\theta}{\cancel{\tan^2\theta}\, \tan\theta}$$

$$= \int \cancel{\sec\theta}\, \frac{1}{\cancel{\cos\theta}} \frac{\cancel{\cos\theta}}{\sin\theta}\, d\theta$$

$$= \int \csc\theta \, d\theta$$

do not need to know this.