Research Statement

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My general research interest include the area of Mathematical finance. Harry Markovitz's (1952) Ph.D. thesis "Portfolio Selection" laid the ground for the mathematical theory of finance. In (1969) Robert Merton introduced stochastic calculus into the study of finance and at the same time Fischer Black and Myron Scholes developed their option pricing formula. Nowadays mathematical finance became one of the fastest growing interdisciplinary science. It draws from the disciplines of probability theory, statistics, scientific computing and partial differential equations to provide models and derive relationships between asset prices, market movements and interest rates.

The focus of my research is on two fundamental problems in mathematical finance: Risk Measurements with application in Derivative Pricing, and Optimal Investment with Risk Constraints. I will describe the contribution I made toward a better understanding of these problems as well as current and future projects.

0.1. **Risk Measures.** A first measurement of the risk of a position will be weather its future value belongs or does not belong to the subset of *acceptable risk*, as decided by a supervisor. There is a natural way to define a measure of risk, by describing how close or how far from acceptance a position is. In their seminal paper on coherent risk measures, Artzner, Delbaen, Eber & Heath (1999) set forth axioms which should be satisfied by risk measures, and called risk measures satisfying these axioms *coherent*. They showed that in a finite probability space every coherent risk measure is characterized by a set of probability measures, the risk associated with a random loss being just its maximal expectation over the set of these measures. A position is *acceptable* if its risk measure is non-positive.

In Larsen, Pirvu, Shreve and Tütüncü (2004), we considers a market with a semimartingale price process. We take as given a finite set of valuation and stress measures, which we call *scenario measures*. With each scenario measure there is an associated floor. These measures and floors determine if a random variable, representing the wealth of an agent at the final time, is acceptable.

The central result of our paper is the characterization of random variables, representing the wealth of an agent at a time prior to the final time, from which the agent can trade to final acceptability. The representation is surprisingly simple and shows, in particular, that if one cannot form a martingale measure as a convex combination of scenario measures, then the final acceptability condition imposes no constraint on the initial wealth. Furthermore we define buyer's and seller's prices for contingent claims. These are like utility indifference prices, except they are based on a concept of "risk-measure indifference." The buyer's and seller's prices thus obtained are within the interval of super-replication and sub-replication prices, and in the case of a complete market, coincide with the expected payoff under the martingale measure.

As a practical application we present a stochastic volatility model in which the scenario measures correspond to different levels to which the volatility reverts. In this example we provide a fairly explicit representation for strategies which trade to acceptability.

0.2. **Optimal Investment with Risk Limits.** Managers limit the riskiness of their traders by imposing risk limits on the risk of their portfolios. Lately Value-at-Risk (VaR) risk measure became a tool used for accomplish this purpose. The increased popularity of this risk measure is due to the fact that VaR is easily understood. It is the maximum loss of a portfolio over a given horizon, at a given confidence level. In spite of its popularity, VaR is also known to poses undesirable characteristics. The resolution is to consider other risk measures : Tail Conditional Expectation (TVaR), and Limited Expected Loss (LEL).

Basak and Shapiro (2001) analyse the optimal dynamic portfolio and wealth-consumption policies of utility maximizing investors who must manage risk exposure using VaR. They find that VaR risk managers pick a larger exposure to risky assets than non-risk managers, and consequently incur larger losses when losses occur.

Cuocco, He and Issaenko (2001) develop a more realistic dynamically-consistent model of the optimal behavior of a trader subject to risk constraints. They assume that the risk of the trading portfolio is reevaluated dynamically, by using the conditioning information, hence the trader must satisfy the risk limits continuously. They find that that the risk exposure of a trader subject to VaR and TVaR risk limits is always lower than that of an unconstrained trader.

"Maximizing Growth Rate under Risk Constraints" is a joint project with my thesis advisor, Steven Shreve, and with Gordan Zitkovic. In this project we start with the model of Cuocco, He and Issaenko (2001). In addition we take into account the dynamic version of LEL risk measure introduced in Basak and Shapiro (2001). We solve the intemporal problem of optimal investment subject to these risk measures. Because it leads to maximization of the growth rate, the logarithmic utility would seem a natural choice for money managers. On the other hand this maximization criterion it is known as being too risky. Therefore it appears natural to study the problem of maximizing the growth rate of a portfolio under risk constraints. Our main finding is that the optimal policy is a projection of the optimal portfolio of an unconstrained "log" agent (the Merton proportion) onto the constrained set, with respect to the inner product induced by the variance-covariance volatilities matrix of the risky assets. Our contribution to the field is **forth** fold.

- We give mathematical rigorous proofs of the results of Cuocco, He and Issaenko (2001). We also extend their findings to a financial market which is not necessarilly complete.

- We manage to get closed form solutions (for the log utility case) even when the constrained set depends on the current wealth level.
- We extend the model from the constant coefficient market to one with random coefficients.
- In order to have a better understanding of optimal policies we perform a scenario analysis. We find that under all scenarios of volatilities and means rates of return of the risky assets, a VaR and a TVaR constrained trader is allowed to take some minimal risk. A LEL constrained trader behaves more conservative and does not exhibit the above behavior.

0.3. **Current and Future Research.** In a joint work with Steven Shreve and Gordan Zitkovic we study the optimal control of the drift of a Lèvy process. We are interested in using fixed-point theory to analyze and provide existence results for a class of Lèvy-process-driven stochastic control problems.

My future research is a natural continuation of the work described above. In particular I am interested in extending our results from Larsen, Pirvu, Shreve and Tütüncü (2004) for the case of infinitely many scenario measures. I am also interested in what extent the findings from "Maximizing Growth Rate under Risk Constraints" project remain valid if one allows the possibility of the jumps in the model.

I enjoy learning new fields and using expertise and experience from one area in another. I consider that one's research interests should be flexible, as research areas grow and change quickly. I look forward to a life of active mathematics research.

References

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